



**Calhoun: The NPS Institutional Archive**  
**DSpace Repository**

---

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

---

1972

# The application of linear superposition to Bleich's two-beam theory.

Rodrigues, Jose Elvio de Assis.

Massachusetts Institute of Technology

---

<http://hdl.handle.net/10945/15997>

---

*Downloaded from NPS Archive: Calhoun*



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

**Dudley Knox Library / Naval Postgraduate School**  
**411 Dyer Road / 1 University Circle**  
**Monterey, California USA 93943**

<http://www.nps.edu/library>

THE APPLICATION OF LINEAR SUPERPOSITION  
TO BLEICH'S TWO-BEAM THEORY

Jose Elvio de Assis Rodrigues



THE APPLICATION OF LINEAR SUPERPOSITION  
TO BLEICH'S TWO-BEAM THEORY

by

Jose Elvio de Assis Rodrigues

B. S., United States Coast Guard Academy  
(1966)

SUBMITTED IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF  
MASTER OF SCIENCE IN  
NAVAL ARCHITECTURE AND MARINE ENGINEERING  
AND MASTER OF SCIENCE IN  
MECHANICAL ENGINEERING



- 2 -

# THE APPLICATION OF LINEAR SUPERPOSITION TO

## BLEICH'S TWO-BEAM THEORY

by

Jose Elvio de Assis Rodrigues

Submitted to the Department of Ocean Engineering on May 12, 1972,  
in partial fulfillment for the degree of Master of Science in  
Naval Architecture and Marine Engineering and Master of Science  
in Mechanical Engineering.

### ABSTRACT

The principle of linear superposition is applied to Bleich's two general solutions pertaining to hull-deckhouse interaction. The resulting longitudinal stress distributions for three separate mathematical models are compared with stress distribution results obtained through the use of finite element techniques. In reference to Bleich's theory, procedures are derived for the determination of the stress distribution at locations away from the center of the hull-deckhouse structure. Analytic and finite element procedures are also described for the determination of the deck stiffness.

The principle of linear superposition is found to be valid for Bleich's theory. Because the theory neglects shear lag effects, its application is recommended only for the center portion of structures with relatively long deckhouses.

Thesis Supervisor: Alaa E. Mansour

Title: Assistant Professor of Naval Architecture



#### ACKNOWLEDGMENTS

I wish to express my appreciation and thanks to Professor Alaa Mansour for his guidance and assistance in this thesis.

A special thanks is also extended to my wife, whose support and understanding during the preparation of this thesis and throughout my whole course of instruction at M.I.T. is greatly appreciated.





TABLE OF CONTENTS

	Page
TITLE PAGE . . . . .	1
ABSTRACT . . . . .	2
ACKNOWLEDGMENTS . . . . .	3
TABLE OF CONTENTS . . . . .	4
LIST OF FIGURES . . . . .	5
INTRODUCTION . . . . .	8
CHAPTER I - BLEICH'S TWO-BEAM THEORY . . . . .	10
CHAPTER II - EXPANSION OF BLEICH'S METHODS . . . . .	24
CHAPTER III - ICES STUDEL METHODS . . . . .	32
CHAPTER IV - MODEL WITH BULKHEADS . . . . .	34
CHAPTER V - MODEL WITH INCREASED $1/B$ . . . . .	59
CHAPTER VI - MODEL WITHOUT BULKHEADS . . . . .	67
CONCLUSIONS AND RECOMMENDATIONS . . . . .	76
APPENDIX A - SAMPLE STUDEL PROGRAM . . . . .	78
APPENDIX B - CALCULATIONS FOR DETERMINING $K$ . . . . .	84
APPENDIX C - SAMPLE BLEICH CALCULATIONS . . . . .	87
BIBLIOGRAPHY . . . . .	91



LIST OF FIGURES

Figure Number	Title	Page
1.1	Two-Beam Model	11
1.2	Two-Beam Free Body Diagram	12
1.3	Shear Distribution at Hull-Deckhouse Connection	14
1.4	Sample Stress Distribution	17
1.5	Deflection Diagram	17
1.6	Hull-Deckhouse Free Body Diagram	21
1.7	Deviation Factors at Center of Structure as Function of Parameter $u$	23
2.1	Longitudinal Variation of Deviation Factor $\phi_1(z)$	26
2.2	Longitudinal Variation of Deviation Factor $\phi_2(z)$	28
3.1	STRUDL Output	33
4.1	Dimensions of Model with Bulkheads	35
4.2	Loading, Shear, and Moment Diagram of Model with Bulkheads	36
4.3	One-Quarter Structure Element and Nodal Numbering Diagram for Outer Shell	37
4.4	Element and Nodal Numbering Diagram for Main Deck and Bottom Plating	38
4.5	Element and Nodal Numbering Diagram for Bulkheads	39
4.6	STRUDL Loading Diagram for Model with Bulkheads	40
4.7	Longitudinal Stress Distribution Along Deckhouse Length	42
4.8	STRUDL Stress Distribution at Amidships for Model with Bulkheads	44



4.9	STRUDL Stress Distribution 35 feet from Amidships for Model with Bulkheads	45
4.10	Deflection of Amidships Structure for Model with Bulkheads	46
4.11	Deflection of Transverse Section 40 Feet from Amidships	47
4.12	Vertical Deflections of Main Deck at Side and Hull-Deckhouse Connection	48
4.13	STRUDL Stress Distribution with Mid-Plane Held Rigid	49
4.14	STRUDL Shear Stress Distribution 2.5 feet Above Hull-Deckhouse Connection for Model with Bulkheads	51
4.15	Symmetrical Structure for Evaluation of K	53
4.16	Comparison of Stress Results at Amidships for Model with Bulkheads	55
4.17	Comparison of Stress Results 15 Feet from Amidships for Model with Bulkheads	56
4.18	Comparison of Stress Results 35 Feet from Amidships for Model with Bulkheads	57
5.1	STRUDL Stress Distribution at Amidships for Model with Increased 1/B	60
5.2	STRUDL Stress Distribution 70 Feet from Amidships for Model with Increased 1/B	61
5.3	Vertical Deflections of Main Deck at Side and Hull Deckhouse Connection for Model with Increased 1/B	62
5.4	Comparison of Stress Results at Amidships for Model with Increased 1/B	64
5.5	Comparison of Stress Results 30 Feet from Amidships for Model with Increased 1/B	65
5.6	Comparison of Stress Results 70 Feet from Amidships for Model with Increased 1/B	66



6.1	STRU DL Stress Distribution at Amidships for Model without Bulkheads	68
6.2	STRU DL Stress Distribution 35 Feet from Amidships for Model without Bulkheads	69
6.3	Vertical Deflections of Main Deck at Side and Hull Deckhouse Connection for Model without Bulkheads	70
6.4	STRU DL Shear Stress Distribution 2.5 Feet Above Hull-Deckhouse Connection for Model without Bulkheads	71
6.5	Comparison of Stress Results at Amidships for Model without Bulkheads	73
6.6	Comparison of Stress Results 15 Feet from Amidships for Model without Bulkheads	74
6.7	Comparison of Stress Results 35 Feet from Amidships for Model without Bulkheads	75





## INTRODUCTION

In 1953 H. H. Bleich presented a paper in the "Journal of Applied Mechanics" entitled "Non-Linear Distribution of Bending Stresses Due to Distortion of the Cross Section." In this paper he derived a viable analytical solution to the problem of hull-deckhouse interaction. However, because of the difficulty involved in evaluating the two-coupled differential equations for real life situations Bleich was only able to present two simplified solutions. One solution to the differential equations was simplified by considering only bending moment loads and neglecting the distributed loads. The other solution considered only distributed loads and neglected bending moment loads. Although these simplifying assumptions are not justified in any real ship, the methods of solution are, in actuality, straightforward and rather simple to obtain. The proposal is that the principle of linear superposition can be applied to Bleich's solution in order to determine the total solution. Even though the title of Bleich's article implies non-linearity, the calculated stress distributions found were actually linear in form with the only discontinuity or break appearing at the main deck level. In order to confirm or disprove this hypothesis, finite element methods using the ICES STRUDL program developed at M.I.T. will be employed to find the total bending stress distribution. A comparison of the linear superposition of the two solutions using Bleich's method and that of the total ICES STRUDL solution should give some indication of the applicability of linear superposition to Bleich's method. If superposition is applicable, a rather simple



analytical solution could then be available for the determination of the actual total bending stress distribution.

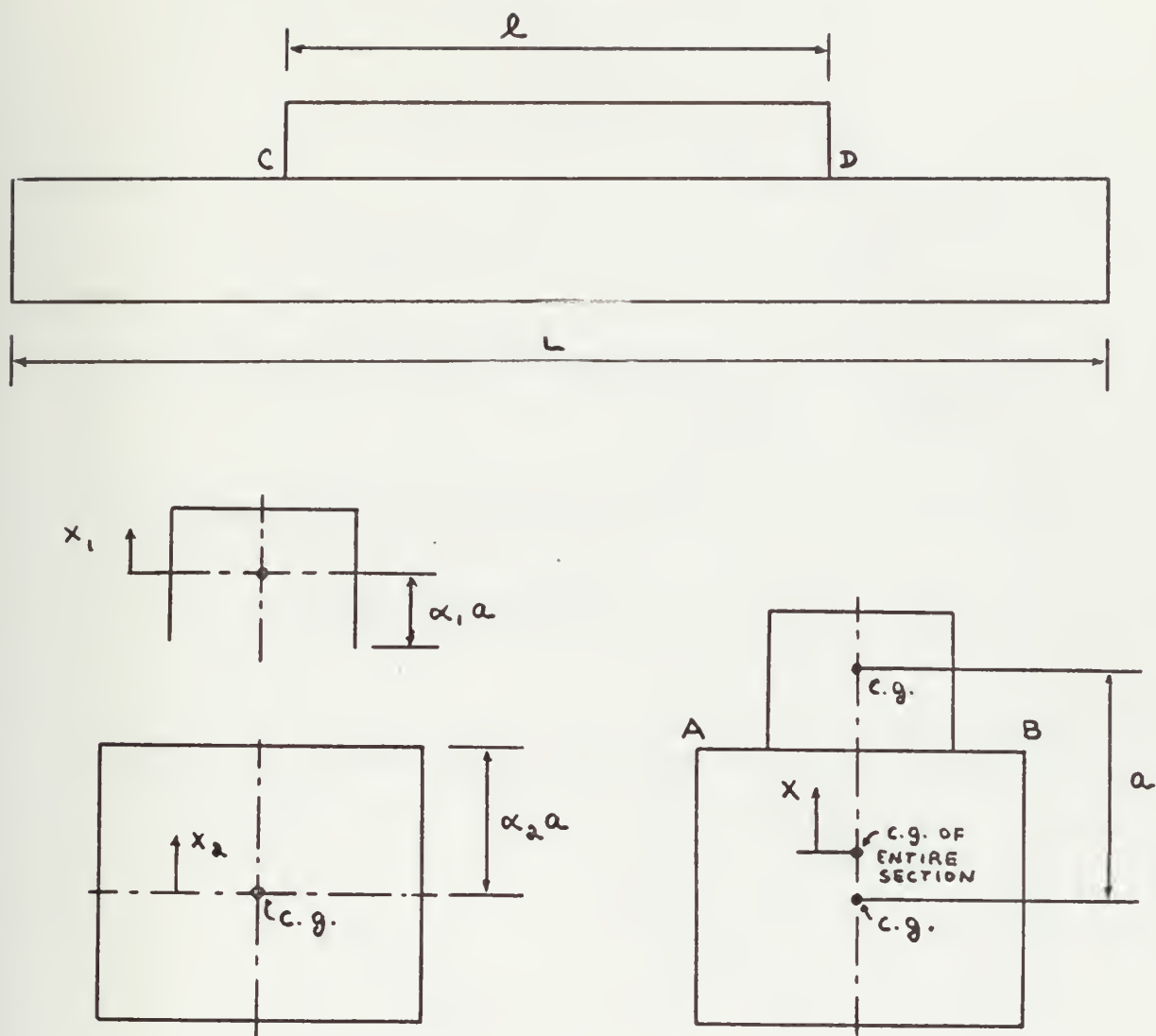


## CHAPTER I - BLEICH'S TWO-BEAM THEORY

Consider the problem of two separate beams forced to act together by horizontal shear forces and vertical forces acting at the junction of hull and deckhouse. In essence, the system consists of a beam elastically supported by another beam with a shear connection to enforce equal strains at the deck level, Figure 1.1 and 1.2. The vertical forces are due to elastic resistance of the deck framing or bulkheads against the motion of the deckhouse with respect to the hull. Navier's hypothesis for a structure acting as a beam is applicable only if the cross sections of the beam do not distort. Since the side plating of a deckhouse is not coplanar with the ship's side, the vertical deflections of the two are not necessarily the same. Navier's hypothesis that plane sections remain plane and the conventional theory of bending of beams are not applicable for the entire structure. For this analysis, however, Bleich assumed that the symmetrical cross section consists of two non-distorting portions which can move relative to each other and that Navier's hypothesis is applicable for the hull and deckhouse each by itself.

In the structure shown in Figure 1.1, the lower hollow box beam represents the hull and is of length  $L$  while the upper box, the deckhouse, is shorter and is of length  $l$ . Both boxes are assumed to be of constant cross section. The cross-sectional area and the moment of inertia of the deckhouse and of the hull are  $A_1$ ,  $I_1$ , and  $A_2$ ,  $I_2$ , respectively, and the distances of the respective centers of gravity from each other and from the deck are  $a$ ,  $\alpha_1 a$ , and  $\alpha_2 a$ .



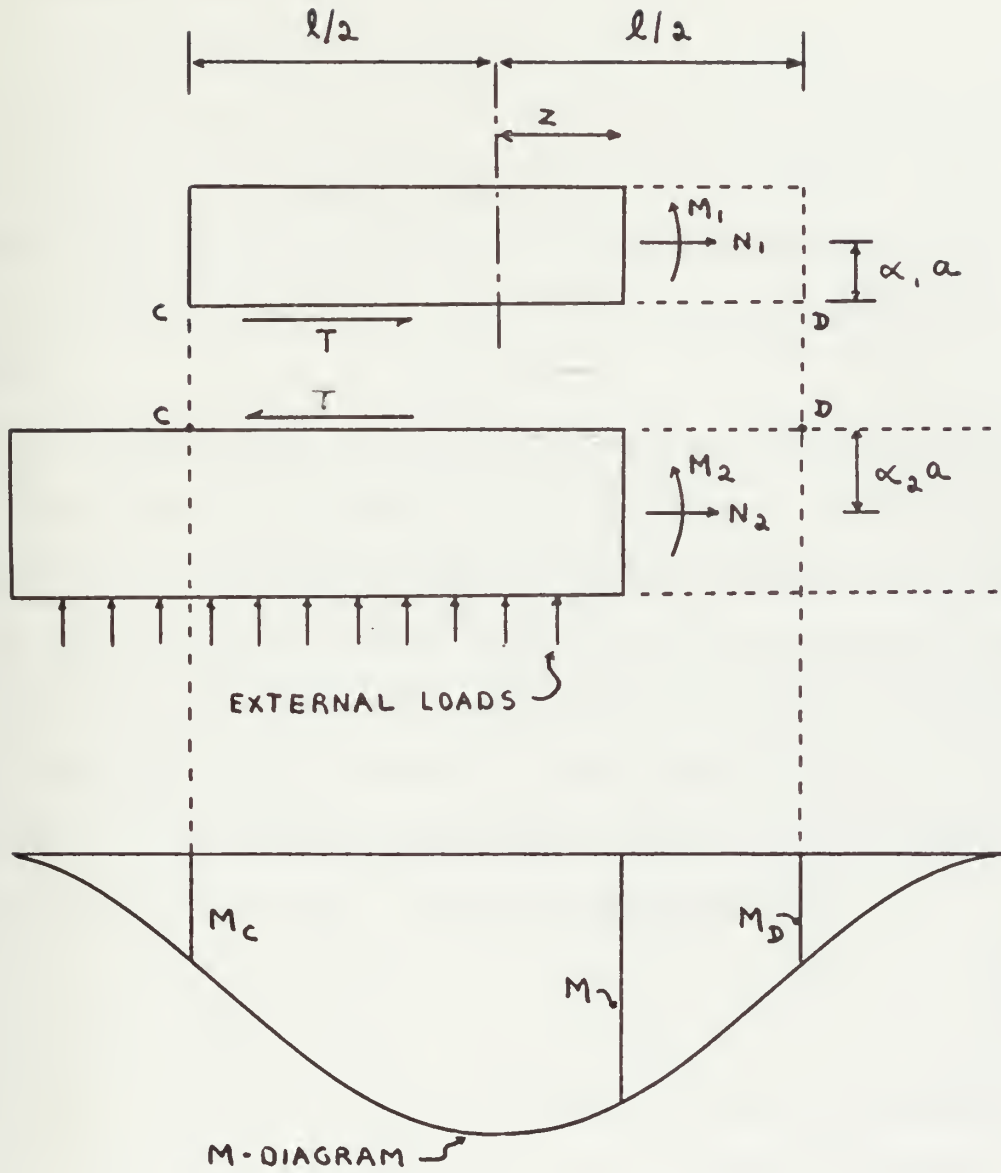


## TWO-BEAM MODEL

FIGURE 1.1







TWO-BEAM FREE BODY DIAGRAM

FIGURE 1.2



## 1. Simplified Analysis of Two-Cell Structure

In this analysis the assumption is made that the deck A-B, figure 1.1, and its supports have no stiffness and, hence, will not resist any relative vertical movements between hull and deckhouse. This simplification is not justified for any real ship system and is discarded in the final analysis; however, some important results are derived.

At a distance  $z$  in the free body diagram of Figure 1.2, the moment and direct forces in the deckhouse and hull are  $M_1$ ,  $N_1$ , and  $M_2$ ,  $N_2$ , respectively, with positive moments producing compression at the top of the respective units. Direct forces  $N_1$  and  $N_2$  are positive if they create tension. The external loads acting to the left of the section have a moment  $M$ . A shear force  $T$  of unknown magnitude will act on the underside of the deckhouse, and a similar force  $T$  will act in the opposite direction on the hull. Equilibrium requires that:

$$N_1 = -T, \quad M_1 = -a\alpha_1 T \quad (1a)$$

$$N_2 = T, \quad M_2 = M - a\alpha_2 T \quad (1b)$$

Since it is assumed that Navier's hypothesis is valid for deckhouse and hull separately, the longitudinal stress distribution across a section can be determined for the deckhouse

$$\sigma_1 = \frac{N_1}{A_1} - \frac{M_1}{I_2} x_1 \quad (2a)$$

and for the hull

$$\sigma_2 = \frac{N_2}{A_2} - \frac{M_2}{I_2} x_2 \quad (2b)$$

with tension stresses counted as positive.



At the junction of the house and hull, the stresses  $\sigma_1$  and  $\sigma_2$  must be equal, with  $x_1 = -a\alpha_1$  and  $x_2 = a\alpha_2$ . Using Equations 1 and 2, the value  $T$  is found to be:

$$T = \frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2 (\alpha_2^2 I_1 + \alpha_1^2 I_2)} M \quad (3)$$

and

$$\frac{dT}{dz} = \frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2 (\alpha_2^2 I_1 + \alpha_1^2 I_2)} V \quad (4)$$

$T$  was defined as the total horizontal shear force acting between the left end of the deckhouse and the section at  $z$ . The total shear force in the structure  $V$  is equal to  $dM/dz$ . According to Equation 3, the shear  $T$  at the ends of the deckhouse is not zero. At points slightly away from the deckhouse ends, there is no deckhouse, and hence  $T = 0$ . This means that in addition to the distributed shear force  $dT/dz$ , there must be concentrated shear forces  $T_c$  and  $T_d$  at the ends of the deckhouse. These concentrated shear forces cannot exist in any real structure. Their appearance is due to the fact that shear-lag effects were neglected when Navier's hypothesis was assumed to be correct for the full length of the deckhouse. These forces will, in reality, distribute themselves over a finite distance, presumably over a distance equal to the depth of the deckhouse as depicted in Figure 1.3.

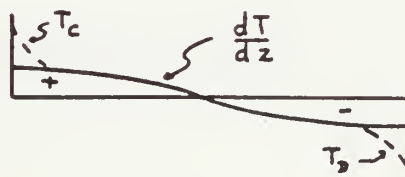


FIGURE 1.3



With this in mind, the stresses found using Equations 2 will be incorrect for calculations near the ends of the deckhouse. According to Saint Venant's theorem, however, the effects of the simplification will not affect the stresses in the center portions of the structure.

In simplifying the expressions for the moments and direct forces, the following notations are used:

$$I_A = a^2 \frac{\Lambda_1 \Lambda_2}{\Lambda_1 + \Lambda_2} \quad (5)$$

$$\mu = \frac{I_1 + \alpha_1 I_A}{I_2 + \alpha_2 I_A} \quad (6)$$

The term  $\mu$  is a measure of the size of the deckhouse in relation to the hull and is referred to as the size factor. The total moment of inertia  $I$  of the section can be expressed in terms of  $I_1$ ,  $I_2$  and by the constant  $I_A$

$$I = I_1 + I_2 + I_A \quad (7)$$

With the use of notations (6-7), the following important expressions are derived:

$$N_2 = -N_1 = T = \frac{MI_A}{aI} - \frac{MI_A}{a} \frac{\mu(\alpha_1 - \mu\alpha_2)}{(1 + \mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (8a)$$

$$M_1 = \frac{MI_1}{I} - MI_1 \frac{\mu}{(1 + \mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (8b)$$

$$M_2 = \frac{MI_2}{I} + MI_2 \frac{\mu^2}{(1 + \mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (8c)$$





These expressions, along with Equations 2, could be used to determine  $\sigma_1$  and  $\sigma_2$  in the deckhouse and hull. It will be noticed, however, that Equations 8 contain two terms the first term being the value of the respective N or M if the conventional bending theory were applicable to the entire section. The actual stresses  $\sigma$  can now be expressed as the sum of the stresses  $\sigma_N$  according to Navier's hypothesis and a correction ( $\Delta\sigma$ ).

$$\sigma = \sigma_N + \Delta\sigma \quad (9)$$

with the Navier's stresses ( $\sigma_N$ )

$$\sigma_N = - \frac{Mx}{I} \quad (10)$$

where x is the distance from the centroid of the entire section.

Figure 1.1. The corrective stresses  $\Delta\sigma_1$  and  $\Delta\sigma_2$  in the deckhouse and hull, respectively, are

$$\Delta\sigma_1 = \frac{\Delta N_1}{A_1} - \frac{\Delta M_1}{I_1} x_1 \quad (11a)$$

$$\Delta\sigma_2 = \frac{\Delta N_2}{A_2} - \frac{\Delta M_2}{I_2} x_2 \quad (11b)$$

where  $\Delta N$  and  $\Delta M$  are the corrective portions of N and M given by the second terms of Equations 8.

Although the model in this analysis was simplified in assuming that no vertical forces act between hull and deckhouse (i.e., zero deck stiffness), the important result remains that the stress distribution can be expressed as the sum of the stress  $\sigma_N$ , according to Navier's theory and the corrective  $\Delta\sigma$ . A typical example of stress distribution using this analysis is shown in Figure 1.4.



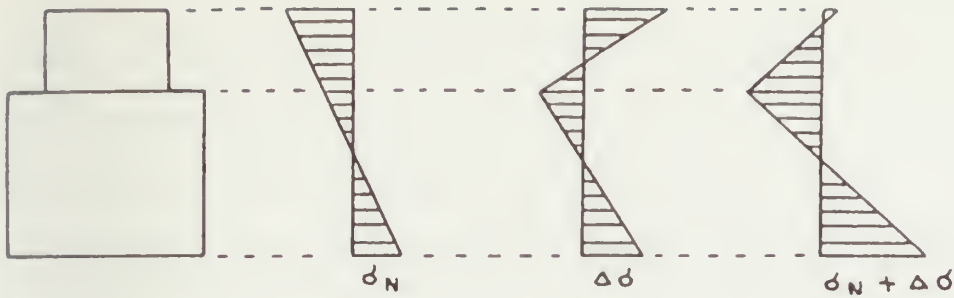


FIGURE 1.4

### General Analysis of Two-Cell Structure

In this analysis the assumption is made that any relative displacement of the deckhouse with respect to the hull is resisted by the internal vertical forces required to deflect bulkheads and transverse beams supporting the deckhouse. External vertical loads and buoyancy will cause the structure to deflect, and this deflection can be described by the displacements  $y_1$  and  $y_2$  of the center lines of the deckhouse and hull, respectively, Figure 1.5.

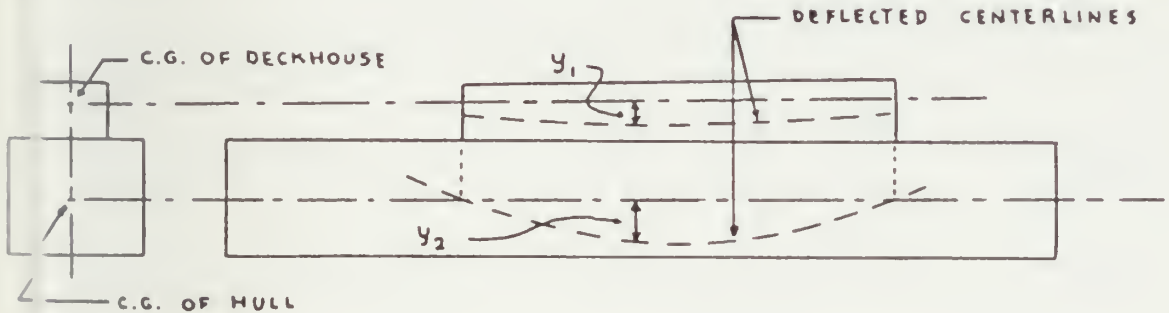


FIGURE 1.5

The stiffness of bulkheads or deck beams resisting relative vertical displacements is assumed to be constant for the full length of the deckhouse, and the magnitude of this stiffness is given by a spring constant  $K$ .  $K$  is defined by Bleich as being the force per unit length



of deckhouse required to produce a relative deflection equal to one unit of length. The vertical reaction between hull and deckhouse will be therefore equal to  $K(y_1 - y_2)$  per unit of length.

Using the theorem of stationary potential energy, the differential equations for the deflections  $y_1$  and  $y_2$  can be obtained. The potential energy  $U$  consists of the internal strain energy and the potential energy of the external forces.

$$\begin{aligned}
 U = & \frac{1}{2} \int_{-\ell/2}^{\ell/2} [EI_1 y_1''^2 + EI_2 y_2''^2 + EI_A (\alpha_1 y_1'' + \alpha_2 y_2'')^2 \\
 & + K(y_1 - y_2)^2 - 2p_1 y_1 - 2p_2 y_2] dz + [y_2']_{-\ell/2}^{\ell/2} \\
 & - [Sy_2]_{-\ell/2}^{\ell/2} .
 \end{aligned} \tag{12}$$

$U$  will be a minimum if the variation

$$\delta U = 0 . \tag{13}$$

Using the calculus of variations, the set of two simultaneous equations are derived.

$$E(I_1 + \alpha_1^2 I_A) y_1^{iv} + Ky_1 + \alpha_1 \alpha_2 EI_A y_2^{iv} - Ky_2 = p_1 \tag{14a}$$

$$\alpha_1 \alpha_2 EI_A y_1^{iv} - Ky_1 + E(I_2 + \alpha_2^2 I_A) y_2^{iv} + Ky_2 = p_2 \tag{14b}$$

$p_1$  and  $p_2$  are the distributed loads acting on the deckhouse and hull, respectively.

The process of the calculus of variations also furnish the boundary conditions required to determine eight arbitrary constants which will appear in the general solutions of the differential equations. At  $z = + \ell/2$  and  $z = - \ell/2$



$$y_2 = 0 \quad (15a)$$

$$E(I_1 + \alpha_1^2 I_A)y_1'' + \alpha_1\alpha_2 EI_A y_2'' = 0 \quad (15b)$$

$$\alpha_1\alpha_2 EI_A y_1'' + E(I_2 + \alpha_2^2 I_A)y_2'' = -M \quad (15c)$$

$$E(I_1 + \alpha_1^2 I_A)y_1''' + \alpha_1\alpha_2 EI_A y_2''' = 0 \quad (15d)$$

The second and third boundary conditions are equivalent to  $M_1 = -\alpha_1 T$  and  $M_2 = M - \alpha_2 T$  as presented in Equations (1). The fourth boundary condition indicates that the shear forces at the end of the deckhouse must vanish.

### 3. Solution of Differential Equations for Constant Moment

In reference to the free-body diagram of Figure 1.6, this analysis neglects the loads  $p_1$ ,  $p_2$  and the shears  $S_C$  and  $S_D$ . The only load on the model is  $M_C = M_D = M$  (i.e., constant moment). The differential Equations 14 are then homogeneous, and since the problem considered is symmetrical with respect to the origin of the coordinate  $z$  axis, the general solution contains four arbitrary constants.

$$y_1 = C_1 + C_2 z^2 + C_3 \sin \gamma z \sinh \gamma z + C_4 \cos \gamma z \cosh \gamma z \quad (16a)$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin \gamma z \sinh \gamma z - \mu C_4 \cos \gamma z \cosh \gamma z \quad (16b)$$

where

$$\gamma = \sqrt[4]{\frac{K}{4E} \frac{1 + \mu}{\alpha_2 I_1 + \mu \alpha_1 I_2}} \quad (17)$$

These solutions in conjunction with the boundary conditions furnish the values of the constants.





$$C_2 = - \frac{M}{2EI} \quad (18a)$$

$$C_3 = \frac{1}{2\gamma^2} \frac{\mu\phi_1 M}{(1+\mu) E(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (18b)$$

$$C_4 = \frac{1}{2\gamma^2} \frac{\mu\psi_1 M}{(1+\mu) E(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (18c)$$

$C_1$  is not included since it only describes rigid body motion. The values of  $\phi_1$  and  $\psi_1$  are

$$\phi_1 = \frac{\sin u \cosh u + \cos u \sinh u}{\sin u \cos u + \sin u \cosh u} \quad (19a)$$

$$\psi_1 = \frac{\cos u \sinh u - \sin u \cosh u}{\sin u \cos u + \sinh u \cosh u} \quad (19b)$$

$$u = \frac{\gamma\ell}{2} = \frac{\ell}{2} \sqrt[4]{\frac{K}{4E} \frac{1+\mu}{\alpha_2 I_1 + \mu\alpha_1 I_2}} \quad (20)$$

Introducing the equations describing the deflections  $y_1$  and  $y_2$  into the following expressions for the moments and direct forces

$$M_1 = - EI_1 y_1'' \quad (21a)$$

$$M_2 = - EI_2 y_2'' \quad (21b)$$

$$N_1 = - N_2 = \frac{EI_A}{a} (\alpha_1 y_1'' + \alpha_2 y_2''). \quad (21c)$$

the following values were found at the amidships section,  $z = 0$ .

$$-N_1 = M \frac{I_A}{aI} - \phi_1 M \frac{I_A}{a} \frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (22a)$$

$$M_1 = M \frac{I_1}{I} - \phi_1 M I_1 \frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (22b)$$

$$M_2 = M \frac{I_2}{I} + \phi_1 M I_2 \frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \quad (22c)$$



A remarkable result is that if a comparison is now made between Equations 8 obtained in Section 1 and Equations 22, the only difference is that the factor  $\phi_1$  is now introduced into the second term. Therefore, the refined theory used in this section furnishes a similar result as that obtained in Section 1 (zero deck stiffness) but modified only by the multiplication of the corrective stress  $\Delta\sigma$  by a factor  $\phi_1$

$$\sigma = \sigma_N + \phi_1 \Delta\sigma \quad . \quad (23)$$

This result is surprisingly simple in that the deviation of the stress distribution from Navier's calculations is determined by a non-dimensional factor  $\phi_1$ . The corrective forces  $\Delta N_1^i$ ,  $\Delta N_2^i$  and moments  $\Delta M_1^i$ ,  $\Delta M_2^i$  are computed from the equations given in Section 1

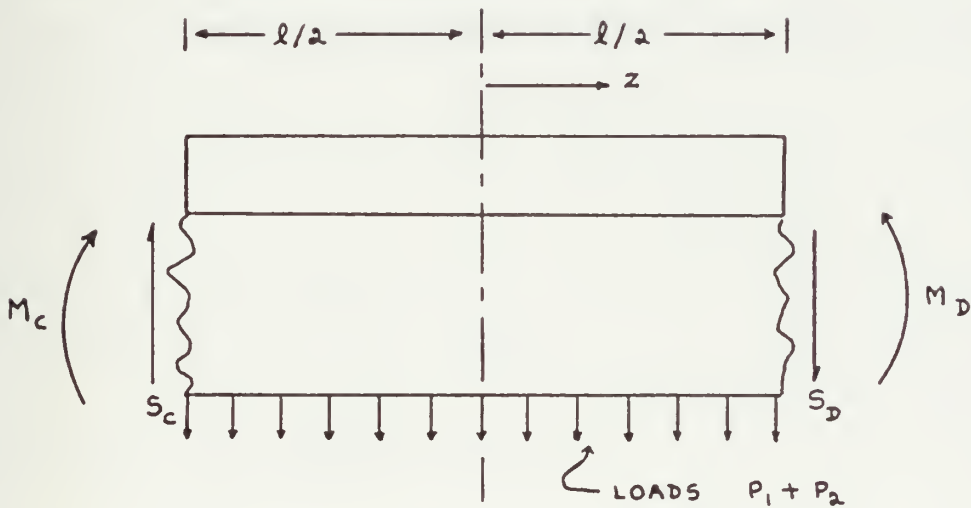


FIGURE 1.6

#### 4. Solution of Differential Equations for Equally Distributed Loads

In this final analysis Bleich considered only the case of equally distributed loads  $p_1$  and  $p_2$  acting on the deckhouse and hull, respectively, where the moments at the ends of the deckhouse are set equal to



zero (i.e.,  $M_C = M_D = 0$ ). Equilibrium requires that the external shear forces  $S_D = -S_C = \lambda/2 (p_1 + p_2)$ . The moment at the center of the section (figure 1.6) due to the loading is

$$M_p = \frac{(p_1 + p_2)}{8} \ell^2. \quad (24)$$

The general symmetrical solution to Equations 14 are

$$y_1 = C_1 + C_2 z^2 + C_3 \sin \gamma z \sinh \gamma z + C_4 \cos \gamma z \cosh \gamma z \\ + \frac{(p_1 + p_2)}{24 EI} z^4 + \frac{p_1}{(1 + \mu)K} \quad (25a)$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin \gamma z \sinh \gamma z - \mu C_4 \cos \gamma z \cosh \gamma z \\ + \frac{(p_1 + p_2)}{24 EI} z^4 + \frac{p_2}{(1 + \mu)K}. \quad (25b)$$

The boundary conditions expressed in Equations (15) furnish the values of the constants

$$C_2 = - \frac{M_p}{2EI} \quad (26a)$$

$$C_3 = \frac{1}{2\gamma^2} \frac{\mu \phi_2 M_p}{(1 + \mu) E(\alpha_2^2 I_1 + \mu \alpha_1^2 I_2)} \quad (26b)$$

$$C_4 = \frac{1}{2\gamma^2} \frac{\mu \psi_2 M_p}{(1 + \mu) E(\alpha_2^2 I_1 + \mu \alpha_1^2 I_2)}. \quad (26c)$$

and where

$$\phi_2 = \frac{2}{u} \frac{\sin u \sinh u}{\sin u \cos u + \sinh u \cosh u} \quad (27a)$$

$$\psi_2 = \frac{2}{u} \frac{\cos u \cosh u}{\sin u \cos u + \sinh u \cosh u}. \quad (27b)$$

The values of the constants are similar to those in the preceding section, and the remaining solution for the stress distribution at



$z = 0$  is the same, the only difference being that a new deviation factor  $\phi_2$  replaces  $\phi_1$  and  $\mu_p$  replaces  $M$ .

$$\sigma = \sigma_N + \phi_2 \Delta\sigma \quad (28)$$

Curves of  $\phi_1$  and  $\phi_2$  are plotted as a function of  $u$  in Figure 1.7.

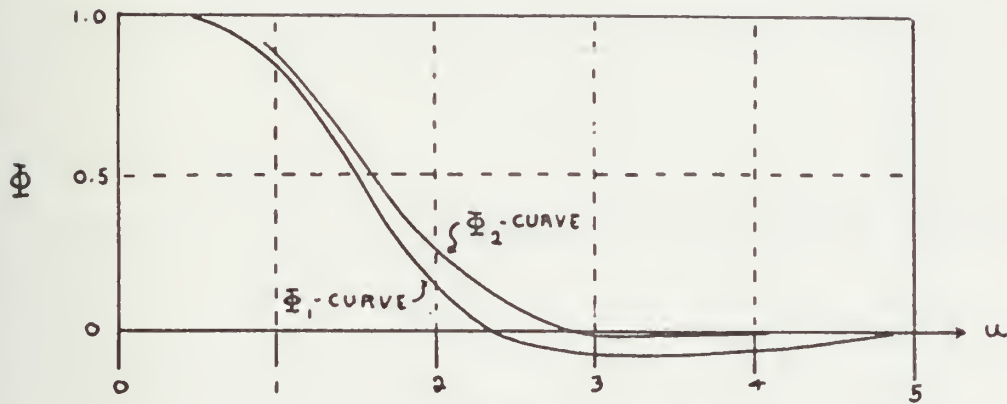


FIGURE 1.7

Since  $u$  is a function of  $K$ , it will be noticed that for the case of zero deck stiffness ( $K = 0$ ) that  $\phi_1$  and  $\phi_2$  are both equal to 1.0, and hence the results of section one are confirmed  $\sigma = \sigma_N + \Delta\sigma$ . When  $K$  or the dimensions of the deckhouse became large and hence  $U > 2$ , the factors  $\phi_1$  and  $\phi_2$  became very small indicating that Navier's stress distribution is applicable at the amidships section.





## CHAPTER II - EXPANSION OF BLEICH'S METHODS

Although Bleich's analysis dealt primarily with the stress distribution located at the center of the deckhouse ( $z = 0$ ), it is the intention of the present analysis to also investigate the stress distributions away from the center. Although Bleich mentions that the relationship for stresses along the deckhouse will be in the form of

$$\sigma = \sigma_{xx} + \phi(z) \Delta \sigma \quad , \quad (1)$$

no expressions for  $\phi$  as a function of  $z$  were provided. They can, however, be readily formulated.

### 1. Constant Moment Loading

For the case of constant moment loading, the expressions for the deflections are:

$$y_1 = C_1 + C_2 z^2 + C_3 \sin yz \sinh yz + C_4 \cos yz \cosh yz \quad (2a)$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin yz \sinh yz - \mu C_4 \cos yz \cosh yz \quad . \quad (2b)$$

The second derivatives of  $y_1$ ,  $y_2$  with respect to  $z$  can be introduced into the expressions for the moments  $M_1$ ,  $M_2$ , and  $N_1$ ,  $N_2$ .

$$M_1 = -EI_1 y_1'' \quad , \quad M_2 = -EI_2 y_2'' \quad (3a)$$

$$N_1 = -N_2 = \frac{EI_A}{a} (\alpha_1 y_1'' + \alpha_2 y_2'') \quad (3b)$$

yielding the following results:

$$M_1 = \frac{MI_1}{I} - [\phi_1 \cos yz \cosh yz - \psi_1 \sin yz \sinh yz]$$

$$MI_1 \frac{\mu}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} \quad (4a)$$



$$N_2 = \frac{MI_2}{I} + [\psi_1 \cos \gamma z \cosh \gamma z - \psi_1 \sin \gamma z \sinh \gamma z]$$

$$MI_2 \frac{\mu^2}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} \quad (4b)$$

$$N_2 = -N_1 = \frac{MI_A}{aI} - [\psi_1 \cos \gamma z \cosh \gamma z - \psi_1 \sin \gamma z \sinh \gamma z]$$

$$\frac{MI_A}{a} \frac{\mu(\alpha_1 - \mu \alpha_2)}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} \quad (4c)$$

These expressions are exactly in the form of Equations 22 of Chapter 1 except that a new deviation factor now exists such that

$$\sigma = \sigma_N + \phi_1(z) \Delta \sigma \quad (5)$$

where

$$\phi_1(z) = [\psi_1 \cos \gamma z \cosh \gamma z - \psi_1 \sin \gamma z \sinh \gamma z] \quad (6)$$

In order to modify the argument of the trigonometric and hyperbolic functions, it will be remembered that  $u = \frac{\gamma \ell}{2}$ , and hence  $\gamma z = 2u \frac{z}{\ell}$ .

Now  $\phi_1(z)$  can be expressed in terms of a non-dimensional distance from the center of the deckhouse.

$$\phi_1(z) = \psi_1 \cos 2u \frac{z}{\ell} \cosh 2u \frac{z}{\ell} - \psi_1 \sin 2u \frac{z}{\ell} \sinh 2u \frac{z}{\ell} \quad (7)$$

With the aid of FORTRAN computer methods to facilitate calculations, values of  $\phi_1(z)$  were calculated as a function of  $u$  and  $(z/\ell)$ . A family of curves for different values of  $u$  is presented in Figure 2.1. In finding the bending stress distribution away from the centerline, no additional calculations need be made other than selecting the appropriate deviation factor from the provided curves and multiplying it by the corrective stress  $\Delta \sigma$ .



# LONGITUDINAL VARIATION OF DEVIATION FACTOR $\Phi_1(z)$

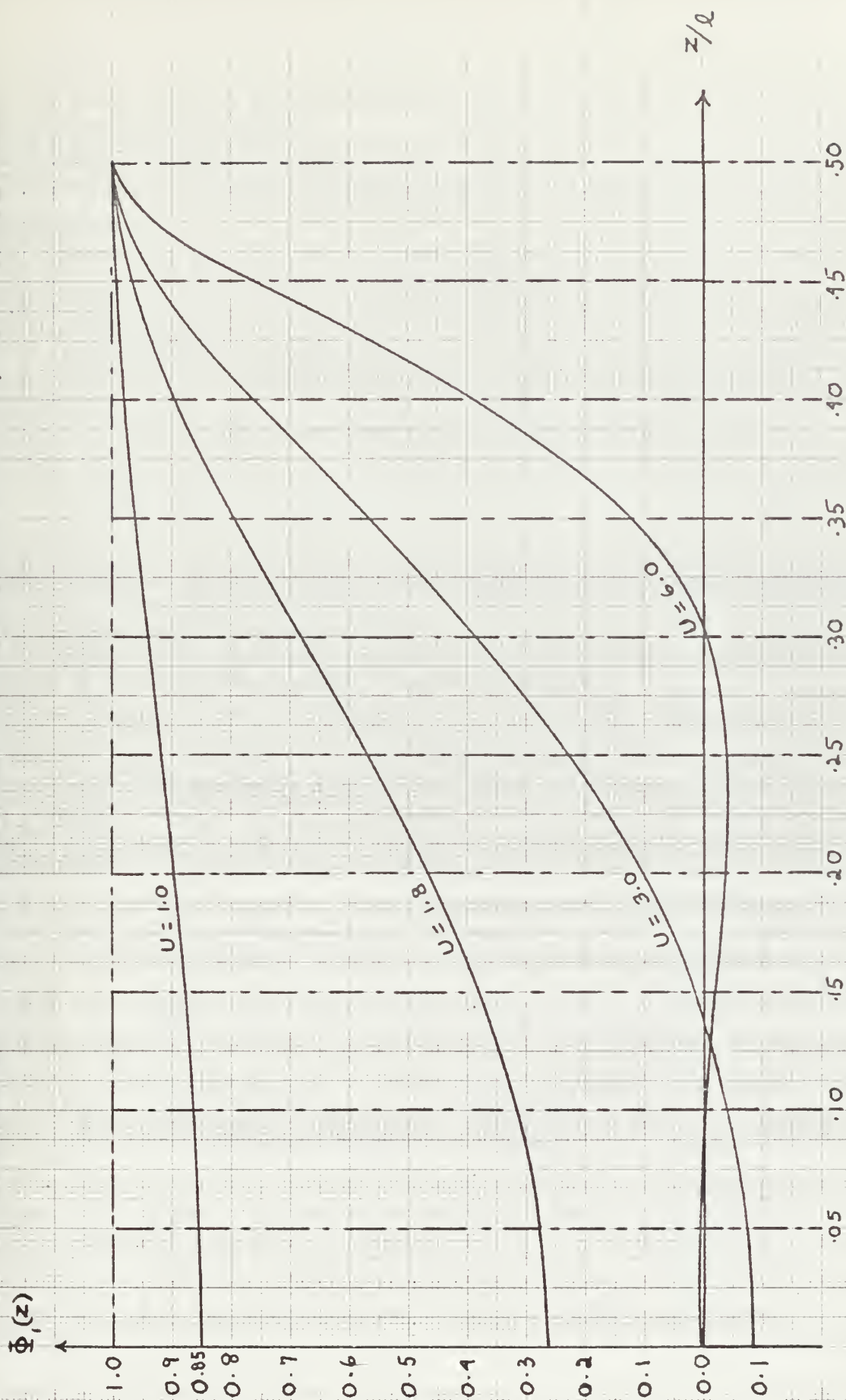


FIGURE 2.1



## 2. Equally Distributed Loads

For the case of equally distributed loads, the equations for the deflections are:

$$y_1 = C_1 + C_2 z^2 + C_3 \sin \gamma z + C_4 \cos \gamma z \cosh \gamma z + \frac{(p_1 + p_2) z^4}{24 EI} + \frac{p_1}{(1 + \mu)K} \quad (8a)$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin \gamma z \sinh \gamma z - \mu C_4 \cos \gamma z \cosh \gamma z + \frac{(p_1 + p_2) z^4}{24 EI} + \frac{\mu p_2}{(1 + \mu)K} \quad (8b)$$

Following the same procedure used in the preceding section, the expressions for the moments and direct forces are:

$$M_1 = \frac{M I_1}{I} - [\phi_2 \cos \gamma z \cosh \gamma z - \psi_2 \sin \gamma z \sinh \gamma z] \\ M_p I_1 \frac{\mu}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} - \frac{(p_1 + p_2)}{2} \frac{I_1}{I} z^2 \quad (9a)$$

$$M_2 = \frac{M I_2}{I} + [\phi_2 \cos \gamma z \cosh \gamma z - \psi_2 \sin \gamma z \sinh \gamma z] \\ M_p I_2 \frac{\mu^2}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} - \frac{(p_1 + p_2)}{2} \frac{I_2}{I} z^2 \quad (9b)$$

$$N_2 = -N_1 - \frac{M I_A}{a I} - [\phi_2 \cos \gamma z \cosh \gamma z - \psi_2 \sin \gamma z \sinh \gamma z] \\ \frac{M_p I_A}{a} \frac{\mu(\alpha_1 - \mu \alpha_2)}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} - \frac{(p_1 + p_2)}{2a} \frac{I_A}{I} z^2 \quad (9c)$$

At first glance these equations do not appear to be in the standard form due to the presence of an additional third term. The first and third terms of each expression can be combined, however, such that





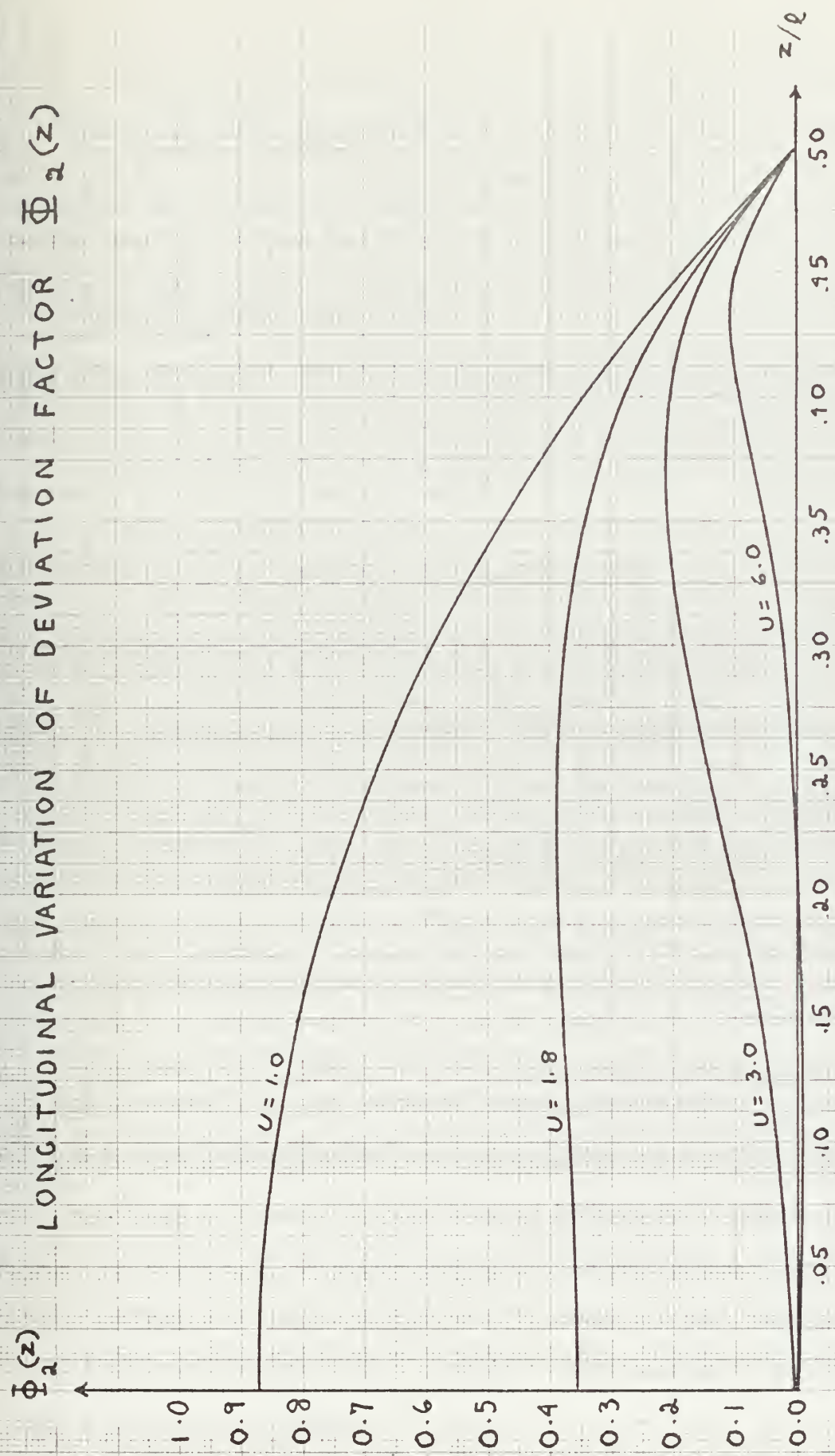


FIGURE 2.2



$$\frac{M_p I_1}{I} - \frac{(p_1 + p_2)}{2} \frac{I_1}{I} z^2 = \frac{M_z I_1}{I} \quad (10a)$$

$$\frac{M_p I_2}{I} - \frac{(p_1 + p_2)}{2} \frac{I_2}{I} z^2 = \frac{M_z I_2}{I} \quad (10b)$$

$$\frac{M_p I_A}{aI} - \frac{(p_1 + p_2)}{2a} \frac{I_A}{I} z^2 = \frac{M_z I_a}{aI} \quad (10c)$$

where  $M_z$  is the moment due to the distributed loads located at a distance  $z$  from the centerline. Equations 9 can then be used to express the stress distribution in the form of

$$\sigma = \sigma_N + \phi_2(z) \Delta\sigma \quad (11)$$

where

$$\phi_2(z) = \phi_2 \cos 2u \frac{z}{\ell} \cosh 2u \frac{z}{\ell} - \psi_2 \sin 2u \frac{z}{\ell} \sinh 2u \frac{z}{\ell} . \quad (12)$$

It must be remembered, however, that the Navier stress term is determined through the use of the moment ( $M_z$ ) at the location in question whereas the corrective stress term ( $\Delta\sigma$ ) employs the use of the moment at the center of the structure ( $M_p$ ).

As in the previous section, computer calculations were used to find  $\phi_2(z)$  as a function of  $u$  and  $(z/\ell)$ . A family of curves for different values of  $u$  is presented in Figure 2.2.

For the case of constant moment loading, the value of  $\phi_1(z)$  at the ends of the deckhouse ( $z/\ell = .5$ ) is equal to 1 for all values of  $u$ . The reverse is true in the distributed load case.  $\phi_2(z)$  is equal to zero at the ends of the deckhouse for all values of  $u$ . This is as expected, since the moments at ends of the deckhouse must vanish in the distributed load model, and, therefore, there can be no corrective stress term at that location.



### 3. Linear Superposition

In reference to obtaining a total solution, Bleich did not mention or suggest the application of linear superposition to his two generalized solutions. On the other hand, there is nothing to indicate that the application of linear superposition is not valid. The simplifying method of breaking the free-body diagram of Figure 1.6 into a constant moment part and a distributed loading part (with its appropriate end shear forces) does not violate equilibrium nor does it alter the actual loading, shear, and bending moment diagrams for the model when the two loading conditions are then summed together. With the assumption that this reasoning also applies to the respective stress distribution solutions, the total stress distribution ( $\sigma_T$ ) will be expressed as the linear sum of the stress due to a constant moment ( $\sigma_M$ ) and the stress due to a distributed loading ( $\sigma_p$ ).

$$\sigma_T = \sigma_M + \sigma_p \quad (13)$$

where

$$\sigma_M = \sigma_M + \phi_1(z) \Delta \sigma_M \quad (14)$$

$$\sigma_p = \sigma_p + \phi_2(z) \Delta \sigma_p \quad (15)$$

Care, however, must be taken such that the appropriate moments are used in these equations.  $\Delta \sigma_p$  for the deckhouse, as an example, employs the use of the moment  $M_p$

$$\Delta \sigma_p = \frac{M_p I_A}{A_1 a} \frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} + M_p \frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} x_1$$

as described by Equations 11 in Chapter 1.  $\Delta \sigma_M$  is different only in that it makes use of the constant moment  $M$  in lieu of  $M_p$ . With this in mind, Equations 14 and 15 may be combined such that:



$$\sigma_T = - \frac{M_x}{I} - \frac{M_z}{I} + \phi_1(z) \Delta\sigma_M + \phi_2(z) \Delta\sigma_p \quad (16)$$

$$\sigma_T = - \frac{(M_x + M_z)x}{I} + \phi_1(z) \Delta\sigma_M + \phi_2(z) \Delta\sigma_p \quad (17)$$

In later chapters this assumption of linearity will be checked by comparison of results on similar models using finite element methods and Bleich's method.





### CHAPTER III - ICFS STRUDL METHODS

The Structural Design Language (STPUDL) developed at the M.I.T. Civil Engineering Systems Laboratory is a series of computer programs for solving structural engineering problems. It is a subsystem of the Integrated Civil Engineering System (ICES) and can be applied in many ways to a wide class of practical problems. An operational system has been implemented on the IBM system /370. When entered into the computer, the STRUDL language is translated by the ICES Command Interpreter, which, for each command, calls upon the appropriate STRUDL program. In essence it is a language with which an engineer can describe a problem, its solution procedures, and ask for results.

Of the various analyses offered in STRUDL, the stiffness analysis was employed for the present investigation. The stiffness analysis is a linear, elastic, static, small displacement analysis.

The basic principle behind finite element techniques is the replacement of the actual physical problem by a model composed of a finite number of discrete elements which are connected at their nodal points. Many elements are available which have different geometric shapes and different applications for the solution of different types of problems. For the present application, the primary element type used is the 'PSR' element. The 'PSR' is a rectangular element with four nodal points, one located at each of the corner joints. Used in conjunction with a plane stress problem, the nodal unknowns or displacements are  $U_1$  and  $U_2$ .



$$U_1 = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$

$$U_2 = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy .$$

No element loads are available, and hence all loads must be in-plane joint loads. While resulting displacements are given at the nodal points, stresses and strains are given only at the baricenter of each element, figure 3.1.

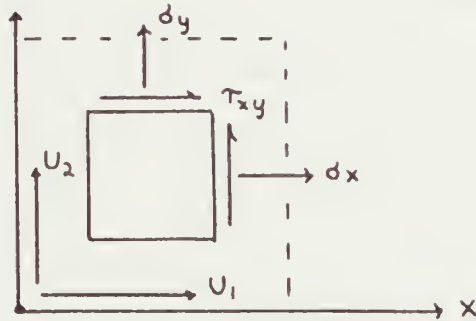


FIGURE 3.1

The material specified must be isotropic, and the only relevant element constants that need be declared are the young's modulus and the Poisson coefficient.

Another element type used was the 'BPR' element. It is used in plate-bending type problems and is similar in geometry to that of the 'PSR' element. The 'BPR' element allows distributed type loads in the out-of-plane direction. In later analyses the 'BPR' elements were superimposed on the 'PSR' elements in order to obtain a more refined analysis.



## CHAPTER IV - MODEL WITH BULKHEADS

For this analysis a model was selected with arbitrary dimensions as shown in figure 4.1. Bulkheads are placed at equally spaced distances of 20 feet in the hull section. The thickness of the hull, box girder plating is .5 inch, and the thickness of the deckhouse plating and bulkheads is .25 inch. The material constants include a Young's modulus of  $30 \times 10^6$  psi and a Poisson coefficient of 0.3.

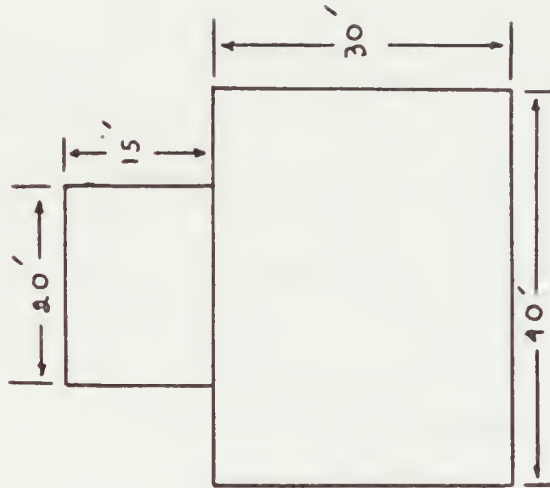
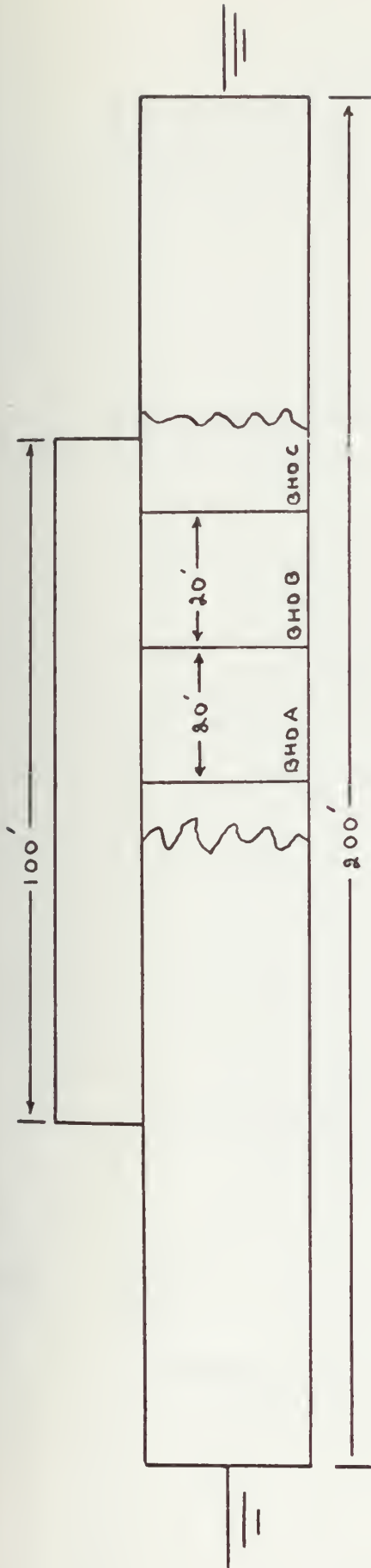
The model is assumed to have a 15-foot draft with a corresponding hydrostatic upward distributed force of 17.143 tons/ft. The internal loading is arranged so as to provide equilibrium and a resultant symmetrical loading, figure 4.2. Shear and moment diagrams for the total model are also provided.

### 1. STRUDEL Model

Due to the symmetry of the loading, it is necessary only to look at one-quarter of the deckhouse and appropriate hull structure. This also aids in reducing computer time which can become excessively high when dealing with finite element methods. The planes of symmetry for the problem are the amidships plane and the centerline plane of the model in figure 4.1. The one-quarter structure, broken into 122 'PSR' elements, is shown in figures 4.3 to 4.5.

Since the 'PSR' elements only allow in-plane joint loads, it was necessary to model the shear, moment, and distributive loads into forces acting on joints or nodal points. The pressure loading acting on the bottom of the hull structure (319.984 psi) was translated into forces acting at bulkhead locations. The moment at the end of the



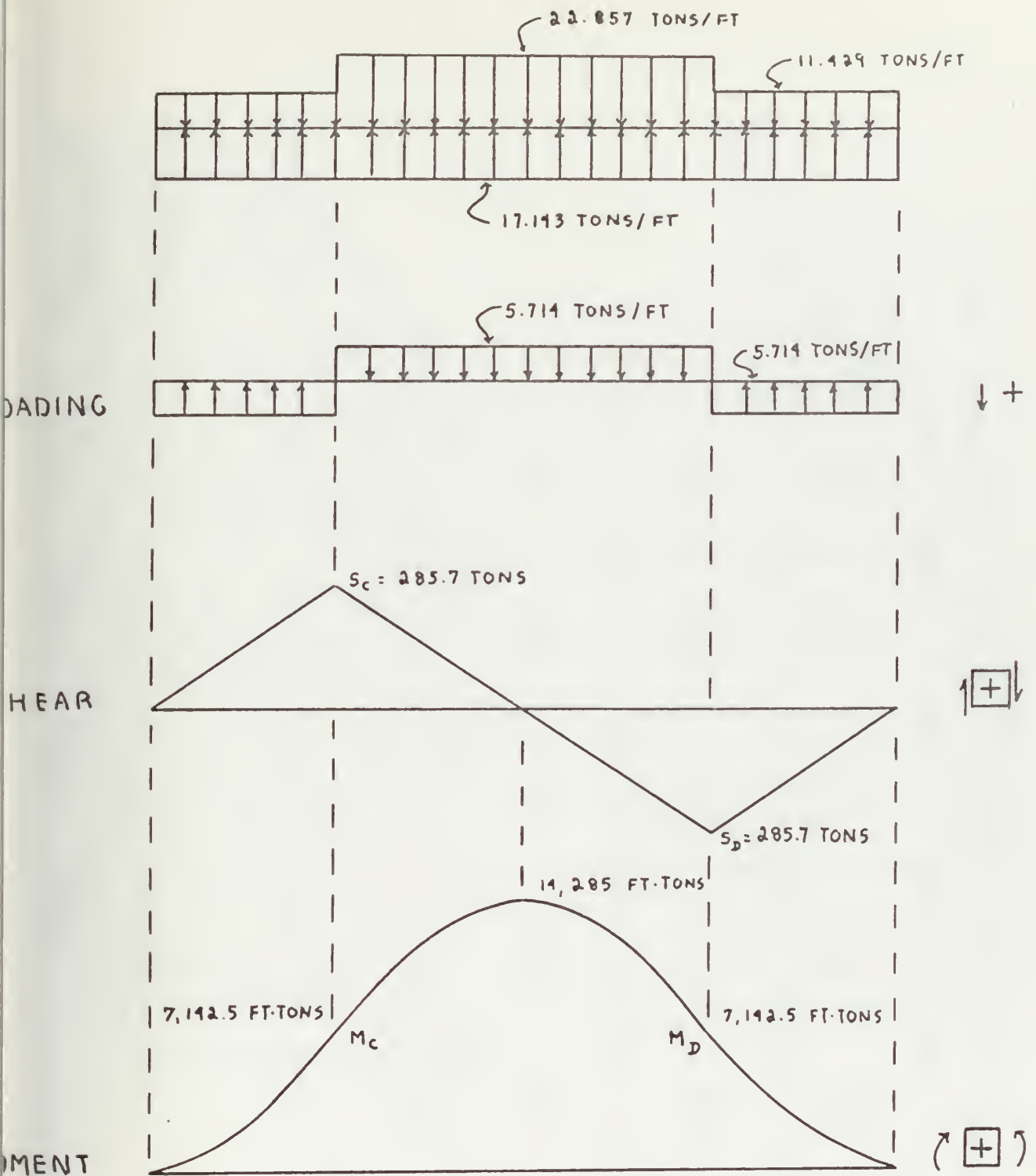


DIMENSIONS OF MODEL WITH BULKHEADS

FIGURE 4.1



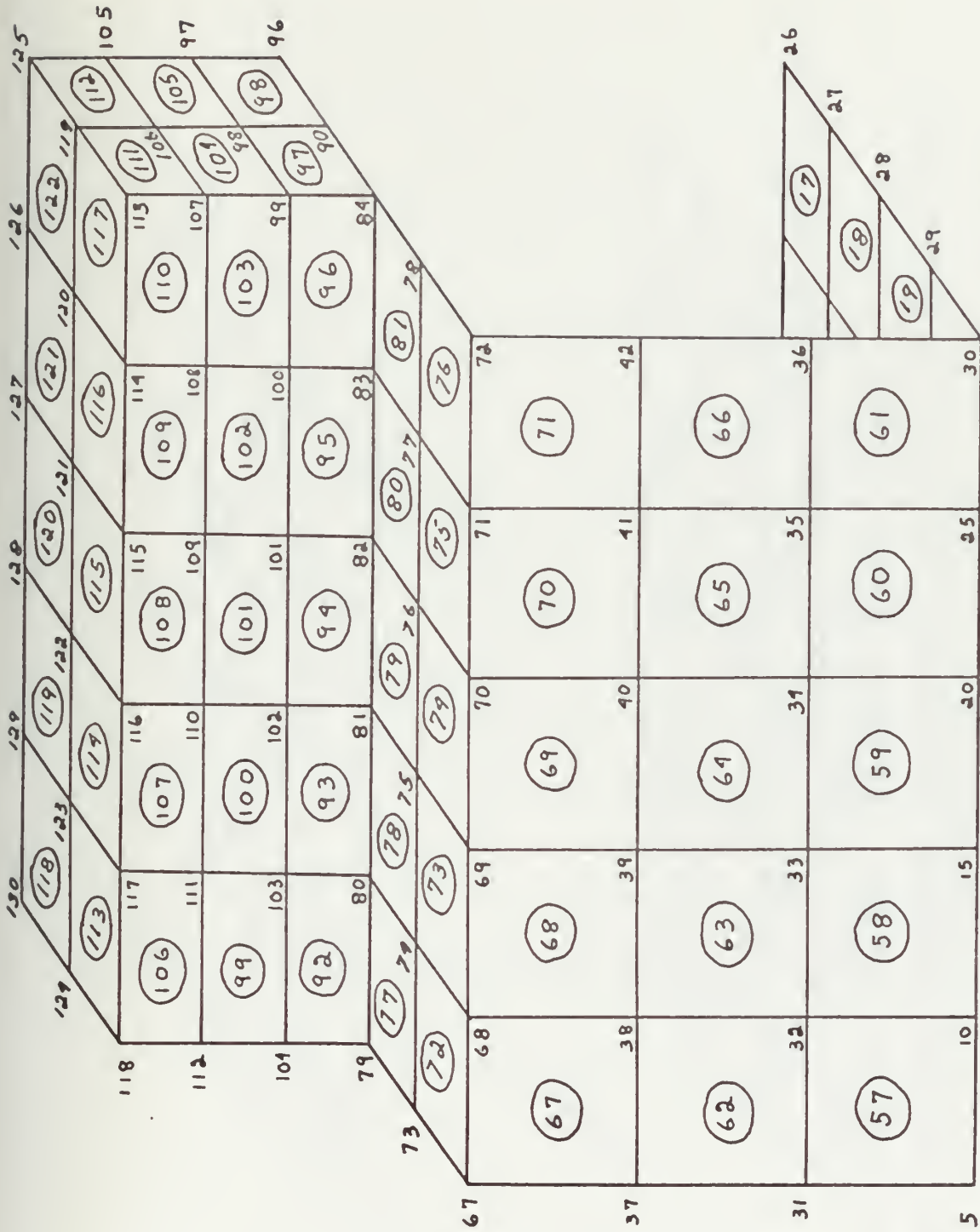




LOADING, SHEAR AND MOMENT DIAGRAM FOR  
MODEL WITH BULKHEADS

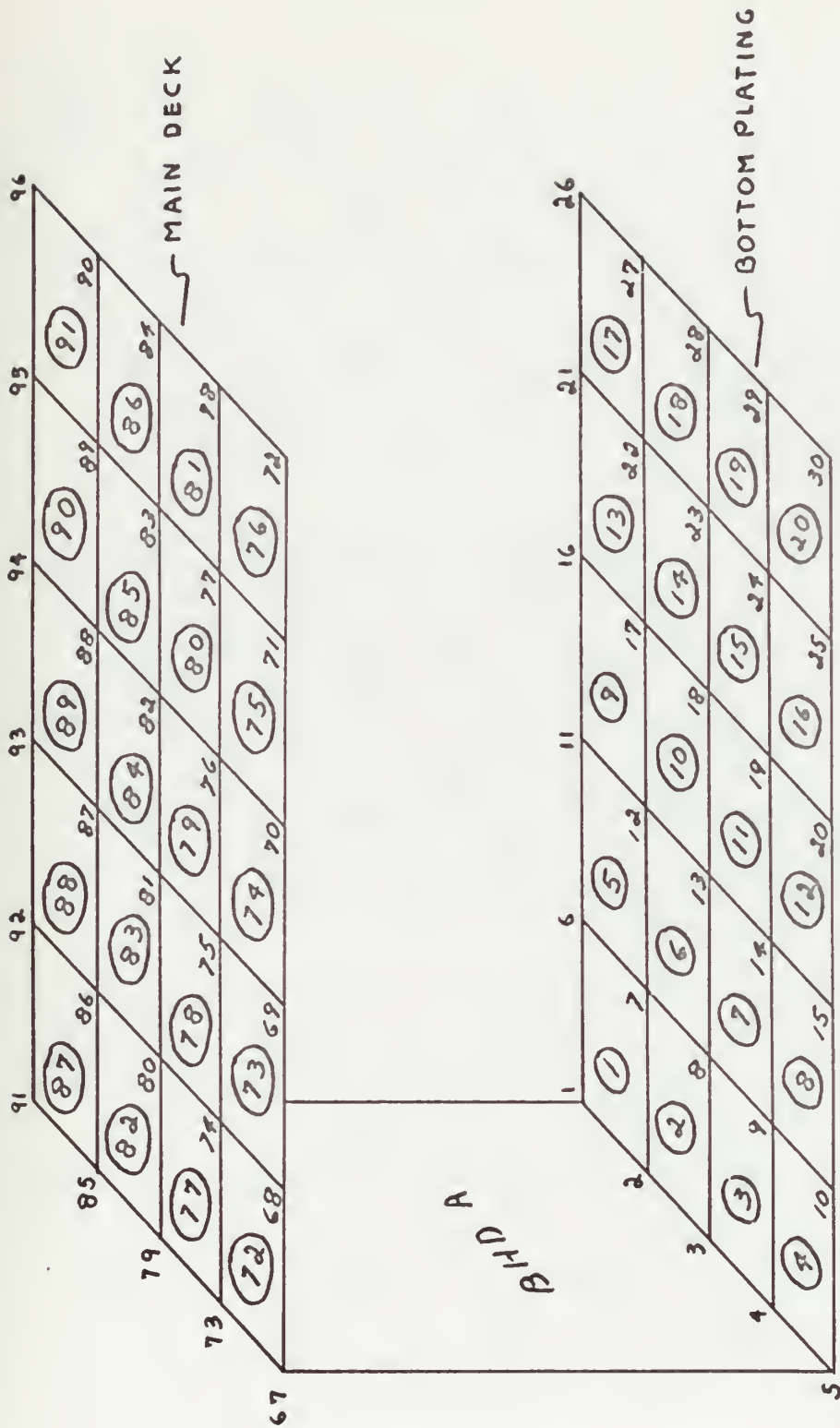
FIGURE 4.2





ONE QUARTER STRUCTURE - ELEMENT AND NODAL NUMBERING DIAGRAM  
FOR OUTER SHELL

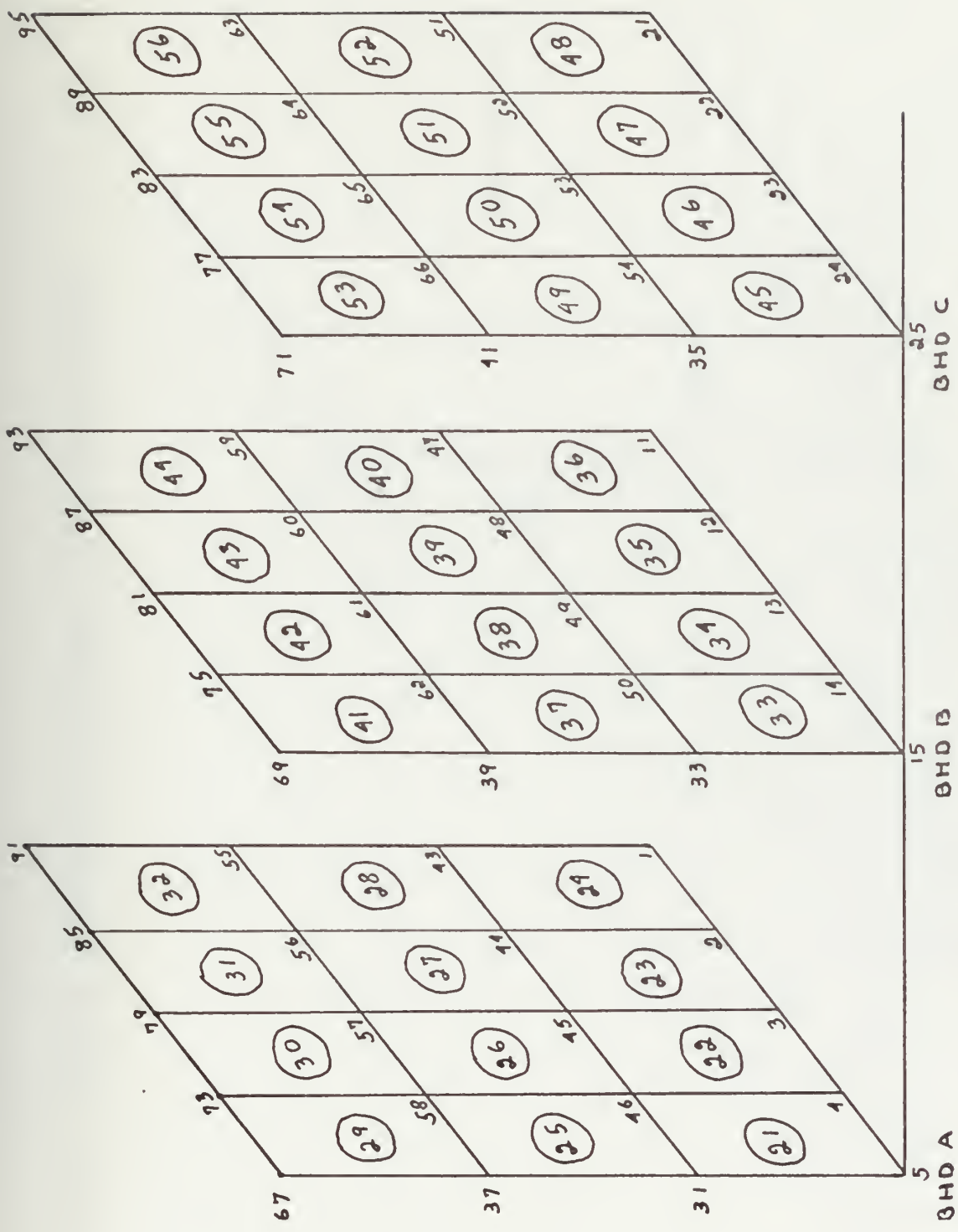




ELEMENT AND NODAL NUMBERING DIAGRAM FOR MAIN DECK AND BOTTOM PLATING

FIGURE 4.4





ELEMENT AND NODAL NUMBERING DIAGRAM FOR BULKHEADS

FIGURE 4.5





## STRUDL LOADING DIAGRAM FOR MODEL WITH BULKHEADS

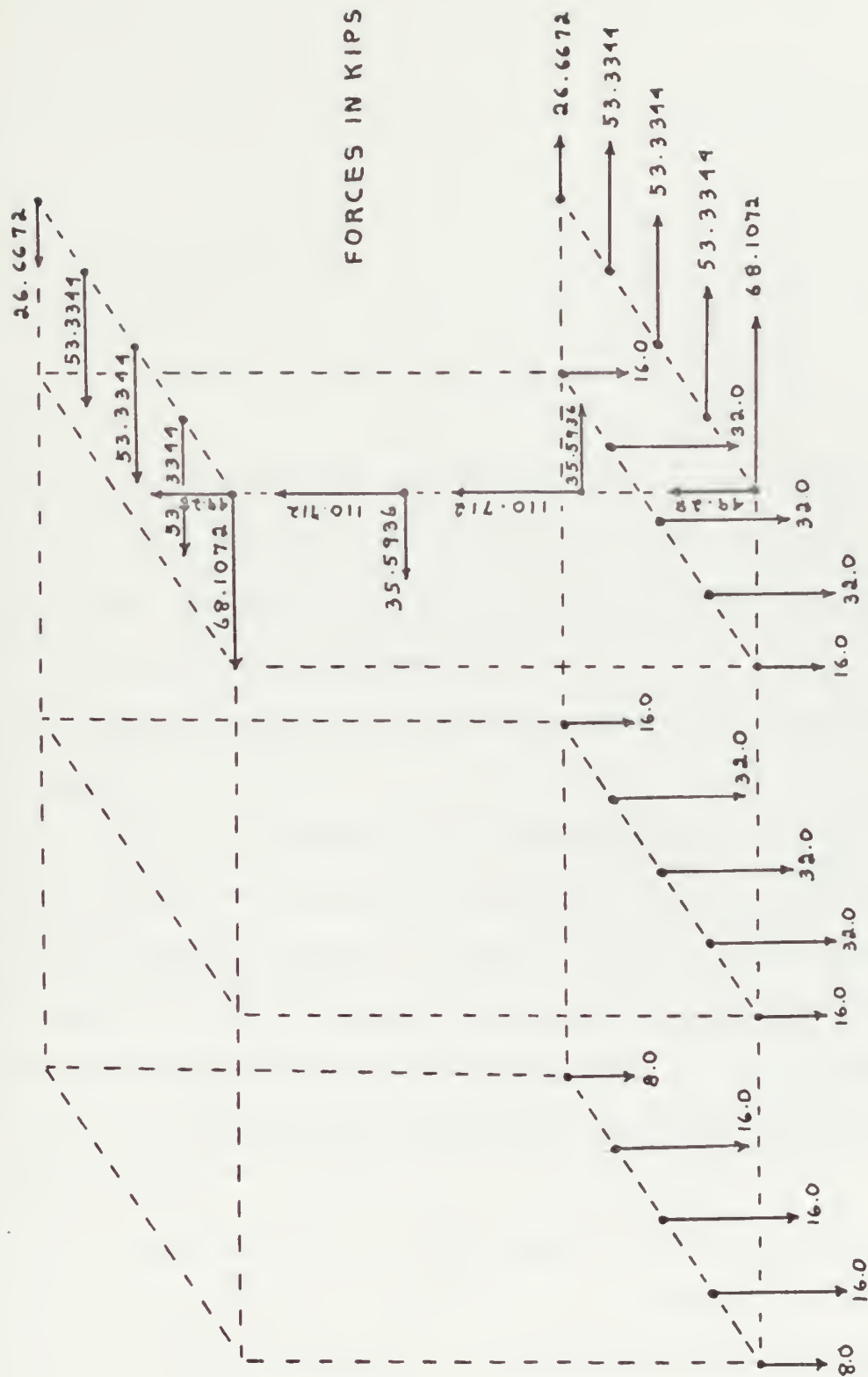


FIGURE 4.6



deckhouse ( $Q_D$ ) was translated into forces acting on the end joints of the hull girder. Using the notation in Section 3.3 of reference 6, the vertical shear flow ( $N$ ) at the end of the structure was found using

$$N = \left[ \frac{3}{2} \frac{Q}{4e} \right] \frac{2Af + a \left[ 1 - \frac{2}{c} \right]^2}{3Af + a} \quad (1)$$

where

$$Q = S_D .$$

The shear flow assumed a parabolic distribution, the maximum occurring at the neutral axis of the hull. The shear flow was then translated into vertical forces acting at nodal points at the end of the hull side. The final loading is depicted in Figure 4.6.

A sample computer program for this loading is provided in Appendix A.

## 2. STRU DL Results

In order to find the magnitude of any distortion due to the application of point loads on the structure, the variation of longitudinal stress along the deckhouse length was plotted, figure 4.7. From these results it appears that some distortion occurs near the deckhouse end as evidenced by the abrupt change in the stress pattern. These effects, however, appear to be concentrated only within a distance of 10 feet of the deckhouse end.

As previously mentioned, the stress output in STRU DL is given at the baricenter of each element. It will be noticed, however, that the baricenters of the elements adjacent to the amidships' location lie on points 5 feet away from amidships. There is little or no stress



WITH BULKHEADS

# LONGITUDINAL STRESS DISTRIBUTION ALONG DECKHOUSE LENGTH

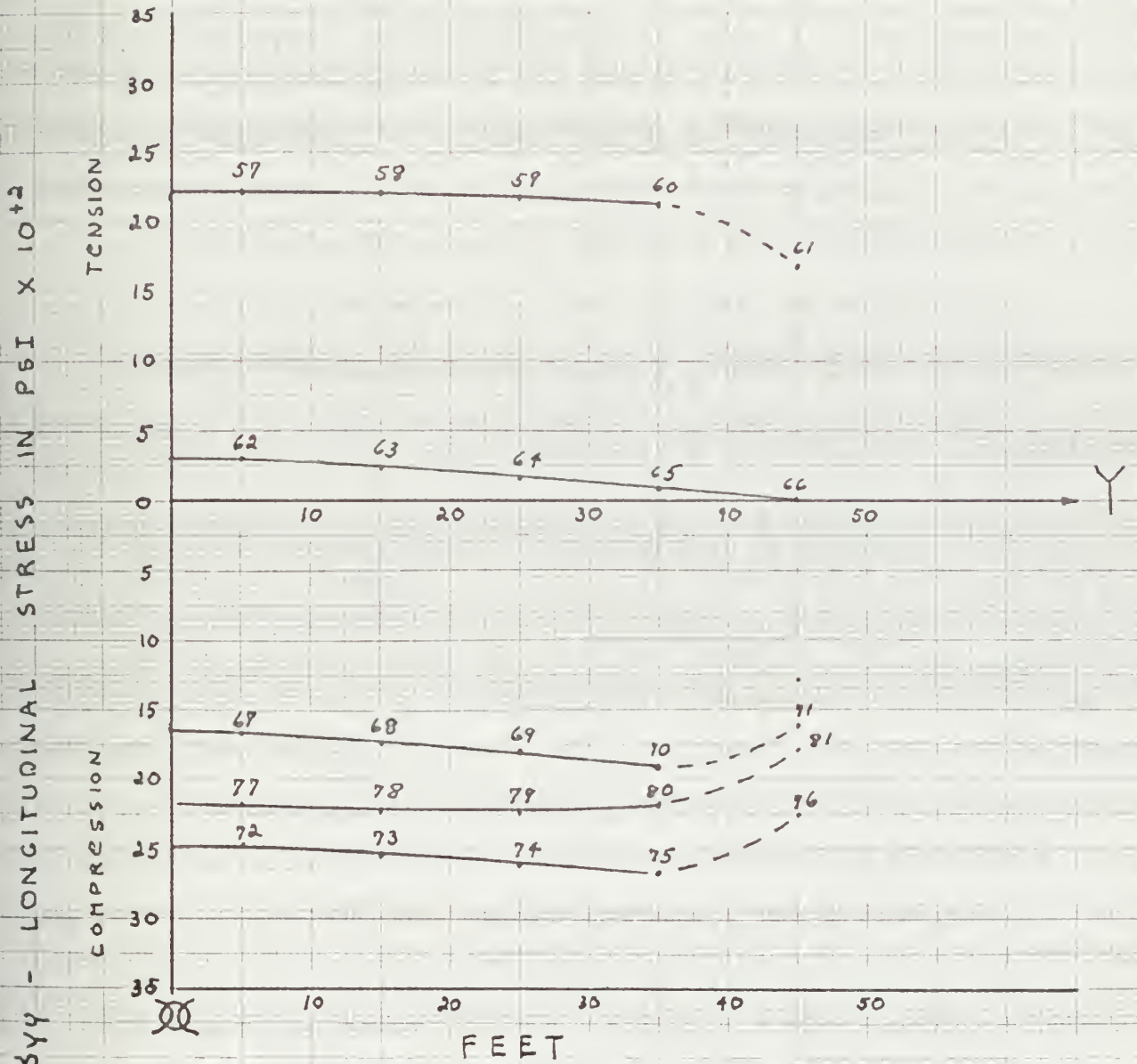


FIGURE 4.7



variation in this area, and hence negligible error is introduced if the stress at the adjacent elements is used for the amidships' results.

The longitudinal stress distributions at amidships and at a point 35 feet from amidships are shown in figures 4.8 and 4.9, respectively. Pronounced shear lag effects are evident in both cases. This is primarily due to the relatively low deckhouse length (l) to hull breadth (B) ratio. For this particular model  $l/B = 2.5$ .

An unexpected result is the discontinuity in the longitudinal stress distribution across the main deck occurring at the junction of hull deck and deckhouse side. An apparent cause for this discontinuity is the nature of the relative deflection of the hull-deckhouse connection with respect to the hull side, figures 4.10 and 4.11. The longitudinal stress due to bending is proportional to the curvature of the beam ( $\frac{1}{\rho}$ )

$$\sigma = - E \left( \frac{1}{\rho} \right) y \quad (2)$$

where  $\rho$  = radius of curvature.

It is apparent that the curves of deflection of the main deck at side and the hull-deckhouse connection as presented in figure 4.12 are quite different from one another. Obviously the curvatures of the two deflections are of different sign or magnitude at respective locations and hence effect the stress distribution across the main deck so as to produce the results shown in figures 4.8 and 4.9. In order to support this reasoning, a separate STRUDL program was run in which the amidships' plane was held rigid. Although this was an unnatural condition, it had the effect of increasing the deck stiffness





# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
ACROSS TRANSVERSE SECTION  
AT AMIDSHIPS  
MODEL - WITH BULKHEADS  
 $L/B = 2.5$

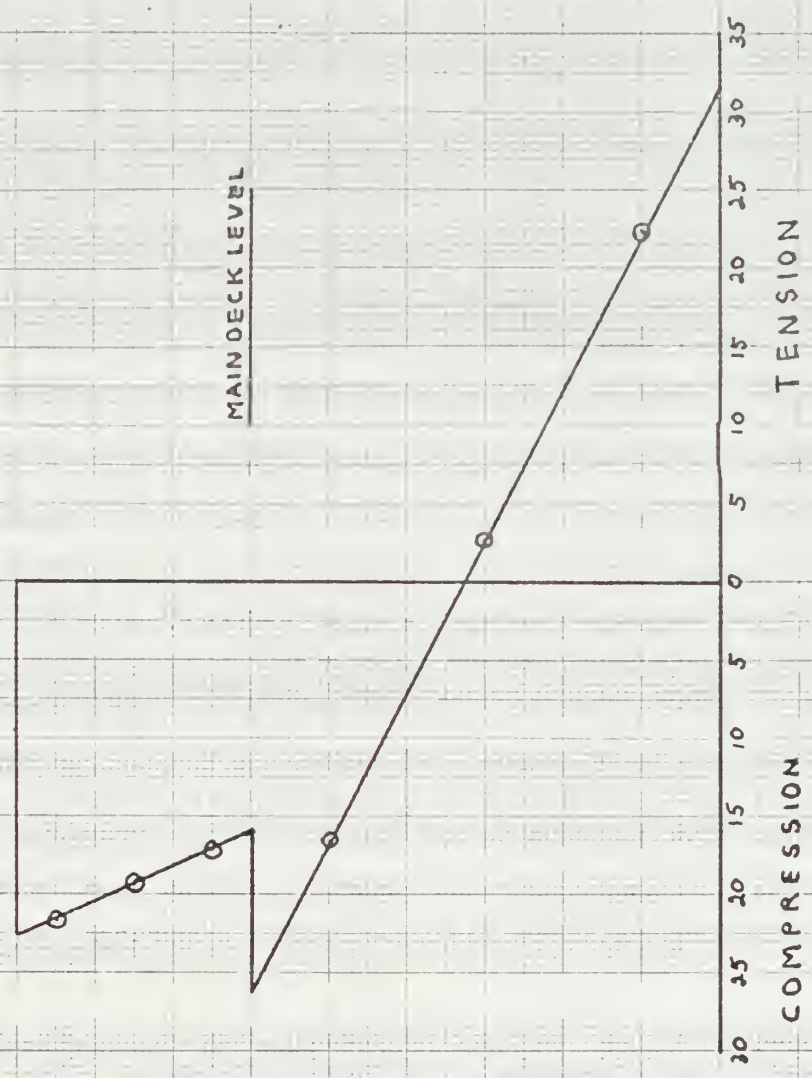


FIGURE 4.8



# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
ACROSS TRANSVERSE SECTION  
35 FEET FROM AMIDSHIPS  
MODEL - WITH BULKHEADS  
 $\lambda/B = 2.5$

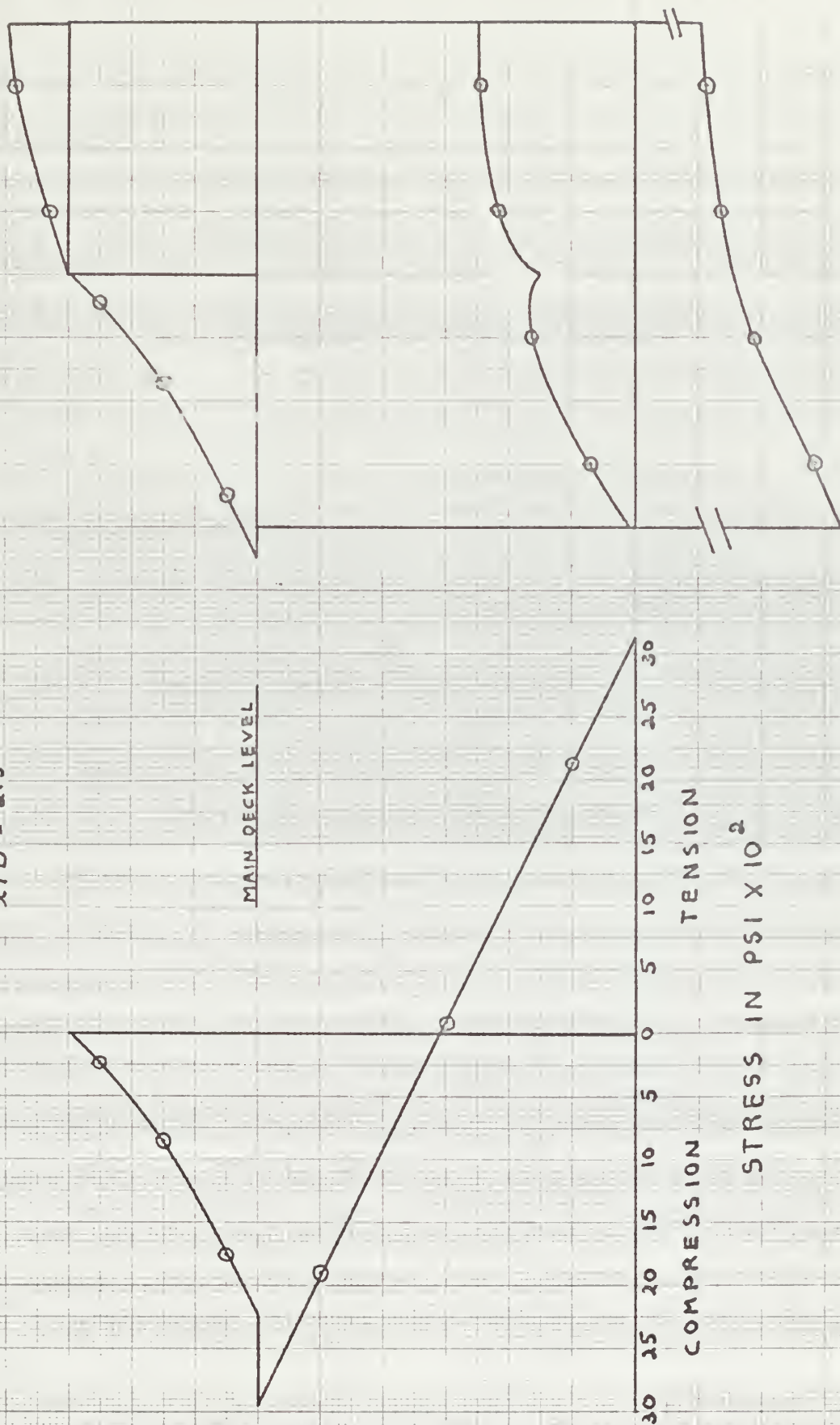


FIGURE 4.9





# STRU DL RESULTS

DEFLECTION OF AMIDSHIPS  
STRUCTURE  
MODEL-WITH BHD'S

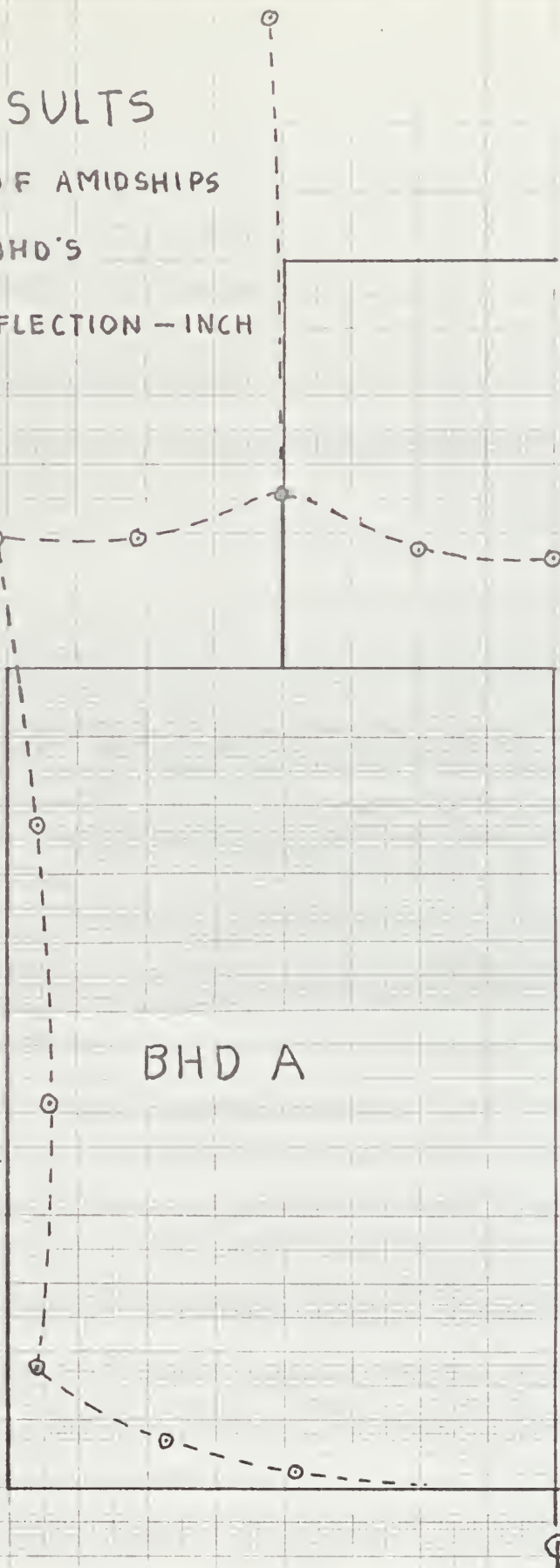
0 .01 .02 .03 DEFLECTION - INCH

STRUCTURE  
AFTER LOADING

STRUCTURE BEFORE  
LOADING

BHD A

FIGURE 4.10





# STRU DL RESULTS

DEFLECTION OF TRANSVERSE  
SECTION 40 FEET FROM  
AMIDSHIPS

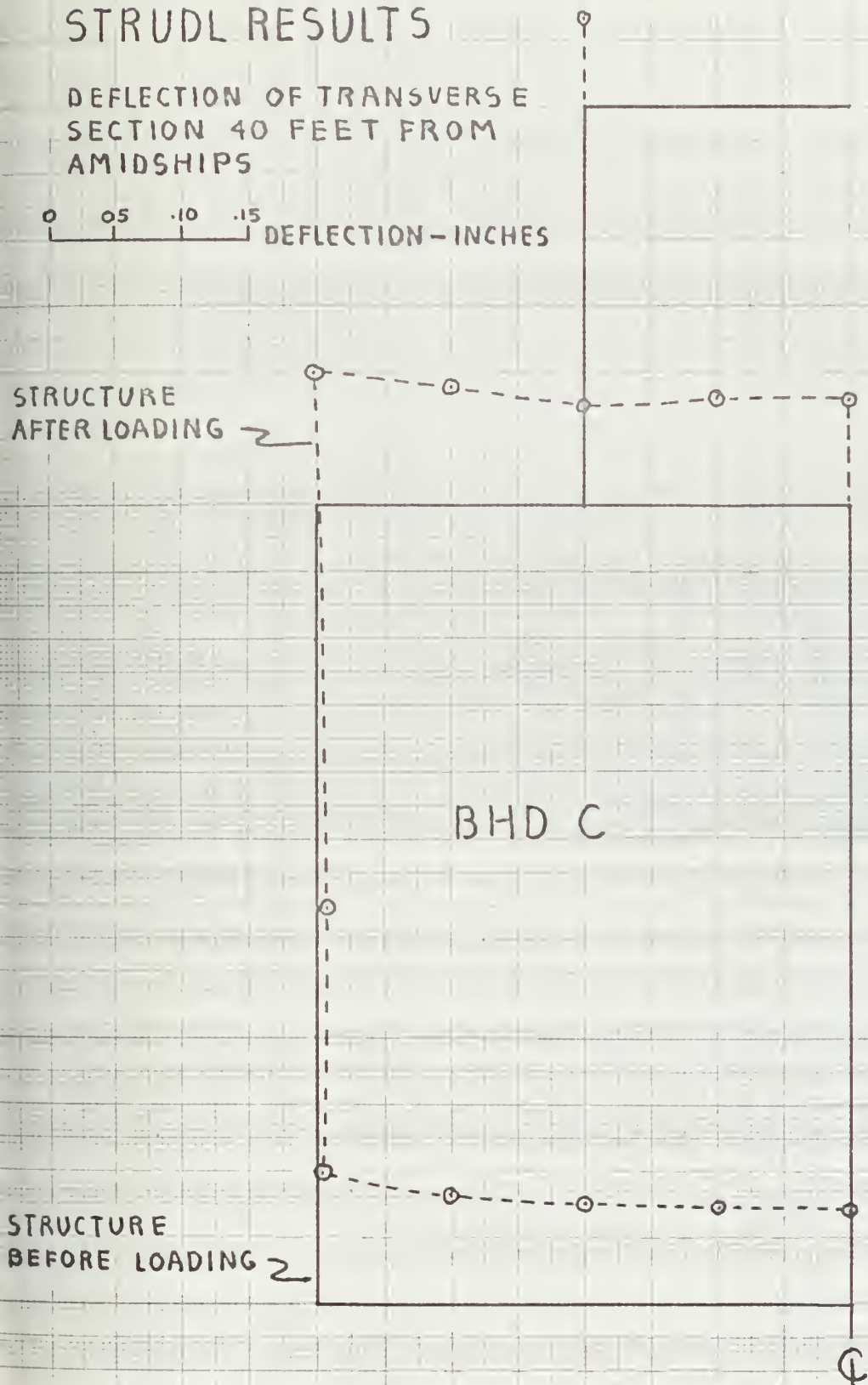
0 0.5 .10 .15 DEFLECTION - INCHES

STRUCTURE  
AFTER LOADING

BHD C

STRUCTURE  
BEFORE LOADING

FIGURE 4.11







# STRUDL RESULTS

## VERTICAL DEFLECTIONS OF MAIN DECK AT SIDE AND HULL-DECKHOUSE CONNECTION MODEL - WITH BHD'S $L/B = 2.5$

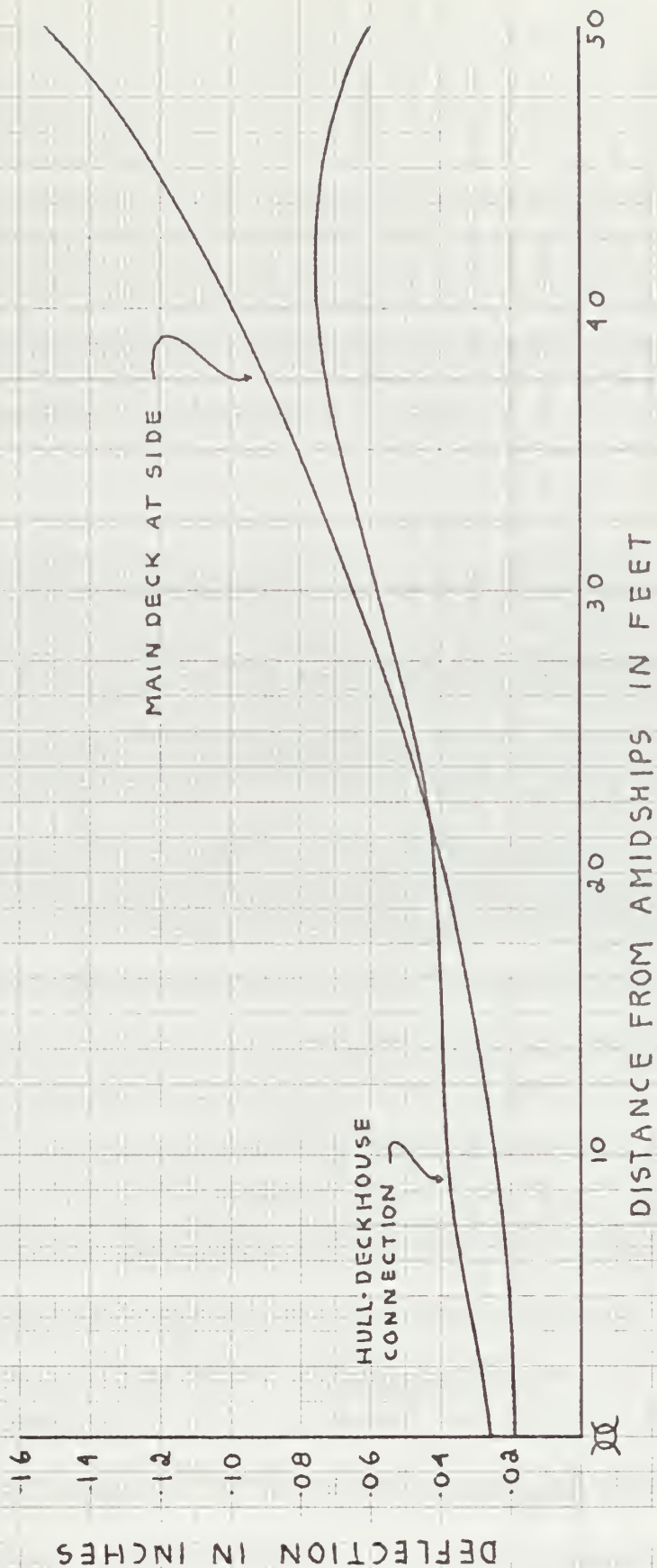


FIGURE 4.12



# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
AT AMIDSHIPS  
MODEL-WITH BHD'S  $L/B=3.5$   
MID-PLANE HELD RIGID

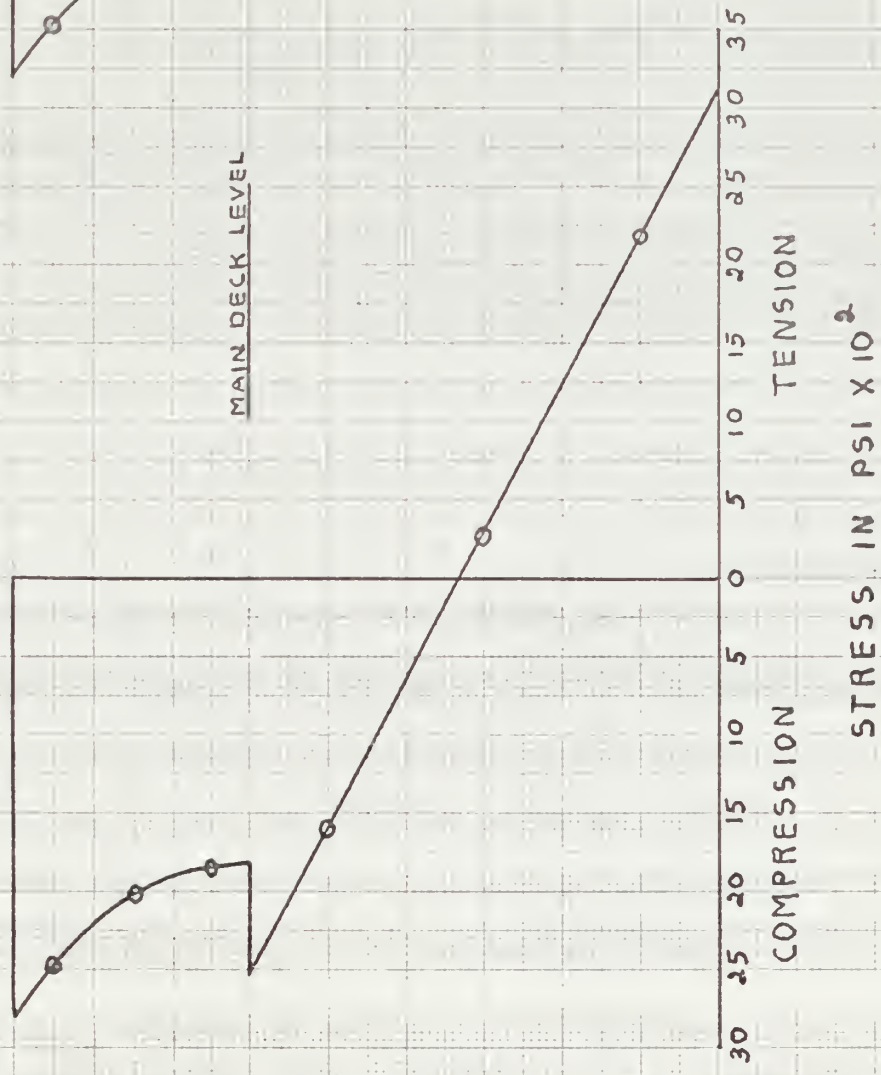


FIGURE 4.13





in the immediate vicinity of the amidships' plane. The hull and deckhouse, therefore, tended to deflect in the same manner, and the discontinuity was virtually eliminated as evidenced in Figure 4.13.

Another area of interest is the distribution of the shear stress along the hull-deckhouse connection. Due to the nature of the elements used, it was only possible to obtain results of the shear-force distribution 2.5 feet above the hull-deckhouse connection. However, the irregular pattern of the shear stress as shown in Figure 4.14 is similar in form to results obtained by experiments on physical models as described in reference 2.

### 3. Bleich Model

The initial step in using Bleich's method is to break the free-body diagram of Figure 1.6 into a constant moment part and a distributed loading part. With reference to Figure 4.2, it can be readily seen that  $M = M_C = M_D = 7,142.5$  ft tons,  $p_1 + p_2 = 5.714$  tons/ft and that  $S_C = S_D = 285.7$  tons. With the use of these values and of the cross structure provided in Figure 4.1, it is a relatively simple procedure to use the solution methods described in Chapters 1 and 2 in order to arrive at a solution.

In a model with bulkheads, however, the primary difficulty is involved with the determination of the deck stiffness or spring constant ( $K$ ). Since  $K$  was defined as the force per unit length of deckhouse required to produce a relative deflection equal to one unit of length, it is apparent that the value of  $K$  will, in reality, vary along the deckhouse length due to the emplacement of structural bulkheads. In order to simplify the use of Bleich's method, however,



# STRUDL RESULTS

SHEAR STRESS DISTRIBUTION  
2.5 FEET ABOVE DECKHOUSE-  
HULL CONNECTION  
MODEL-WITH BHD'S  $L/B = 2.5$

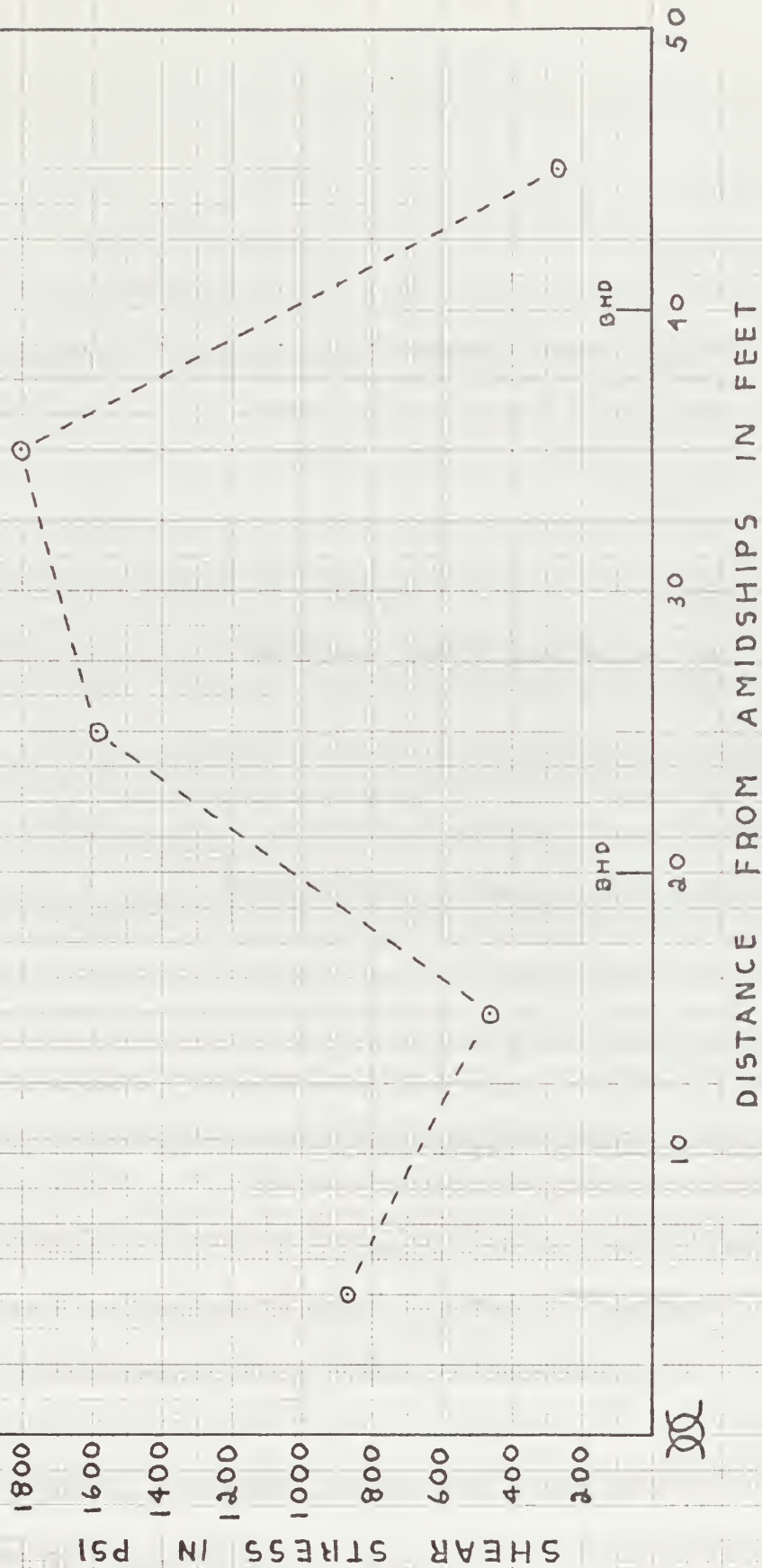


FIGURE 4.14





an average value of  $K$  must be determined. To achieve this end a symmetrical portion of the hull structure was modelled to include a bulkhead and attached deck and bottom plating, figure 4.15. This I-beam type structure is simply supported on its ends and allowed to deflect under vertical forces applied to the hull-deckhouse connections. In the analytical procedure used for determining the forces needed to deflect the hull-deckhouse connections 1 inch, it must be kept in mind that although the deflection due to shear forces are negligible in most cases, in short deep metal beams the deflection caused by shear may become a significant portion of the total deflection. In this case shear contributes a major portion to the total deflection.

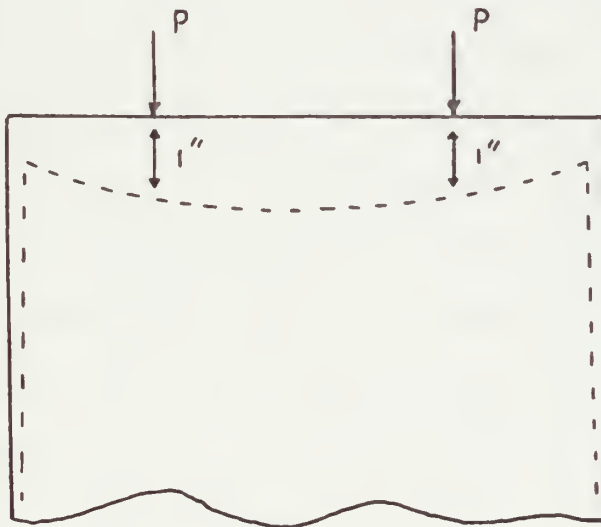
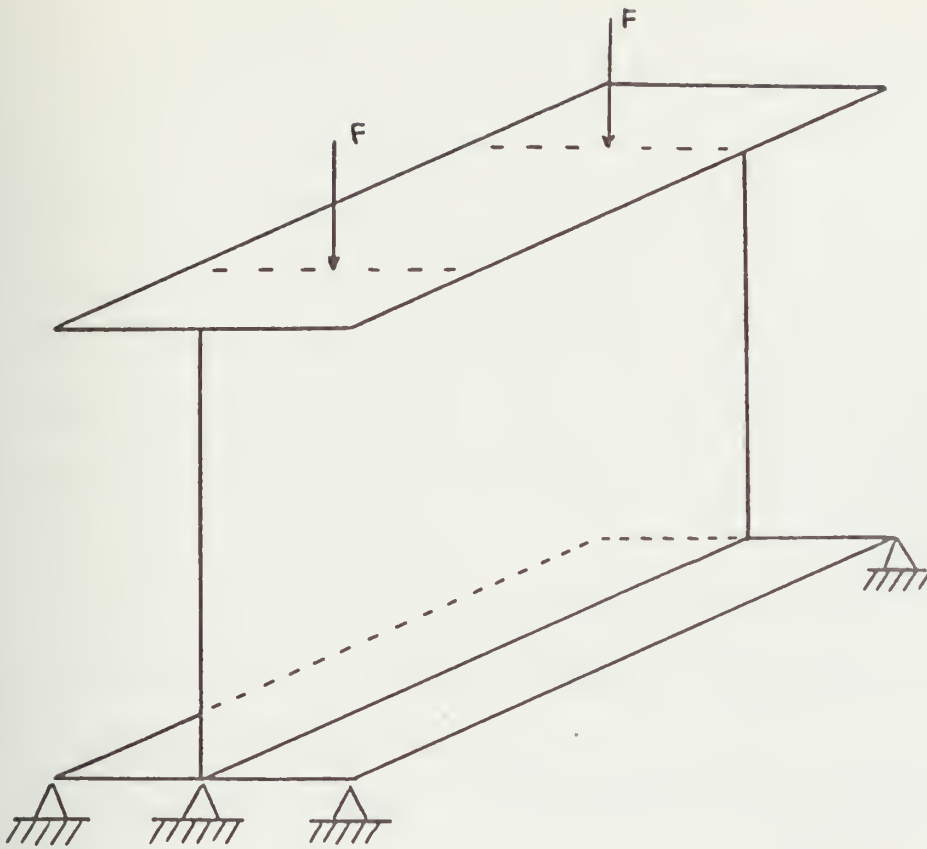
The equation for the deflection due to the internal moments ( $Y_M$ ) was calculated through the use of singularity functions. The expression for the deflection due to shear ( $Y_T$ ) was obtained through the use of the method of unit loads as described in reference 7. The total deflection ( $Y_T$ ) was then expressed as the sum of  $Y_M$  and  $Y_T$  and set equal to 1 inch. The equation was then solved for  $P$  (the distributed load acting on the hull-deckhouse connections). The expression for  $K$  is twice the value of  $P$

$$K = 2P . \quad (3)$$

This analytic approach to  $K$  yielded a value of  $1.86 \times 10^4$  psi for the present model under consideration. Sample calculations are provided in Appendix B.

As a further check on  $K$  a STRUDL program using 'PSR' elements on the model presented in figure 4.15 was run. Arbitrary forces ( $f$ )





SYMMETRICAL STRUCTURE FOR EVALUATION OF  $K$

FIGURE 4.15



were applied at the locations indicated, and the resulting deflection at the hull-deckhouse connection noted. The force was then scaled for a deflection of 1 inch.  $P$  was obtained by dividing  $F$  by the width of the flange (240 inches). Using Equation 3,  $K$  was found to have a value of  $1.53 \times 10^4$  psi. The disparity between the STRUDL  $K$  and the analytical  $K$  can be attributed to the fact that in the analytical approach the deflection calculations apply to the neutral axis of the beam only.

Sample calculations for the solution of the stress distribution at amidships using Bleich's method is provided in Appendix C.

#### 4. Comparison of Methods

Comparison of Bleich and STRUDL results at amidships, 15 feet from amidships and 35 feet from amidships, are shown in figures 4.16, 4.17, and 4.18, respectively. The results for the stress distribution in the vicinity of the deckhouse center compares more favorably than do the results at a location 35 feet away. This is due to the fact that Bleich's solution neglects shear-lag effects, as discussed in Section 1 of Chapter 1. At each location the Bleich stress distribution across the main deck and hull bottom approximates an average value of the STRUDL distribution. The Bleich distributions along the hull side produce lower peak values of stress as compared to STRUDL. The distributions of the stress along the deckhouse side and top attributed to Bleich's method are, in general, of greater magnitude than that of the STRUDL results. The stress distributions for both methods were transformed into moments and checked against equilibrium



# COMPARISON OF METHODS

MODEL-WITH BULKHEADS  $L/B=2.5$   
STRESS DIST. AT AMIDSHIPS  
 $K = 1.53 \times 10^4$

— STRUDL  
- - - GLEICH

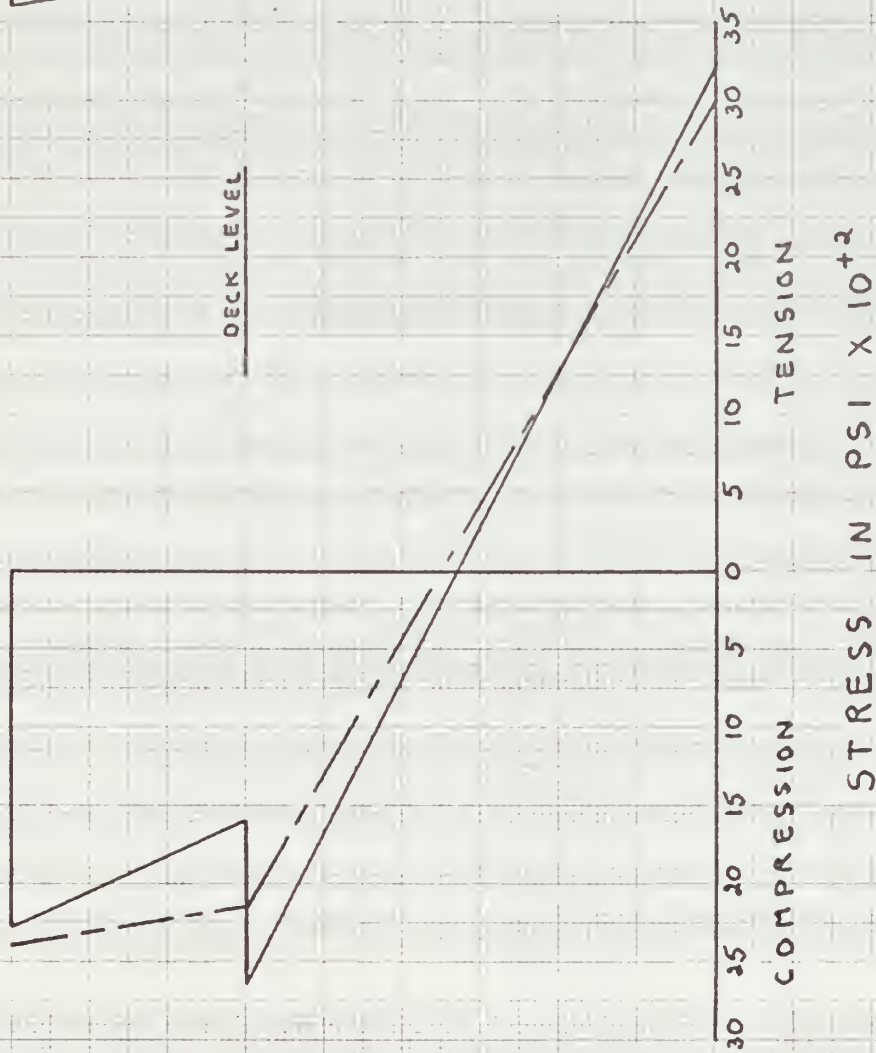


FIGURE 4.16





# COMPARISON OF METHODS

MODEL-WITH BULKHEADS  $L/B = 2.5$   
 STRESS DIST. 15 FEET FROM AMIDSHIPS  
 $K = 1.53 \times 10^4$

— STRUDL  
 - - - BLEICH

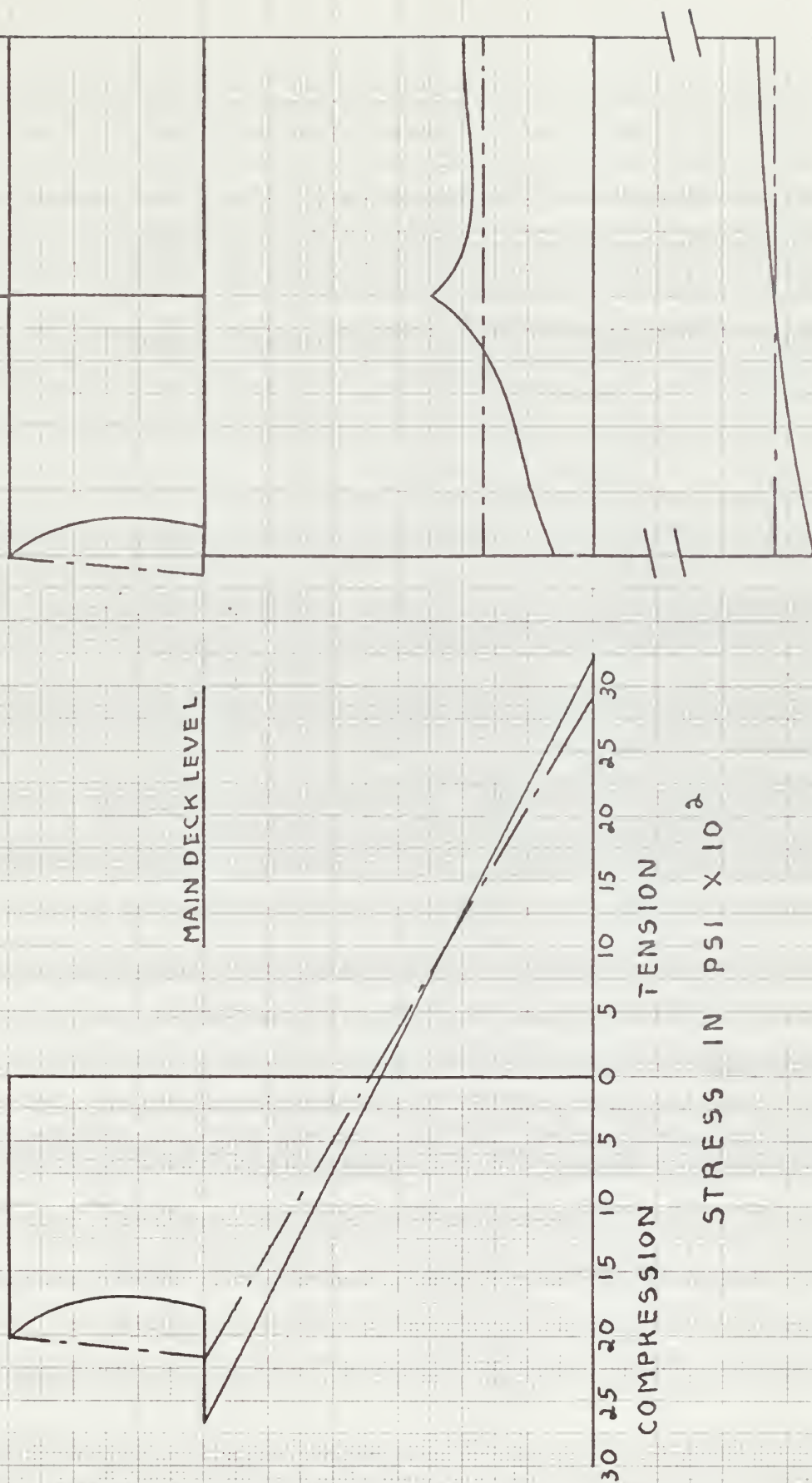


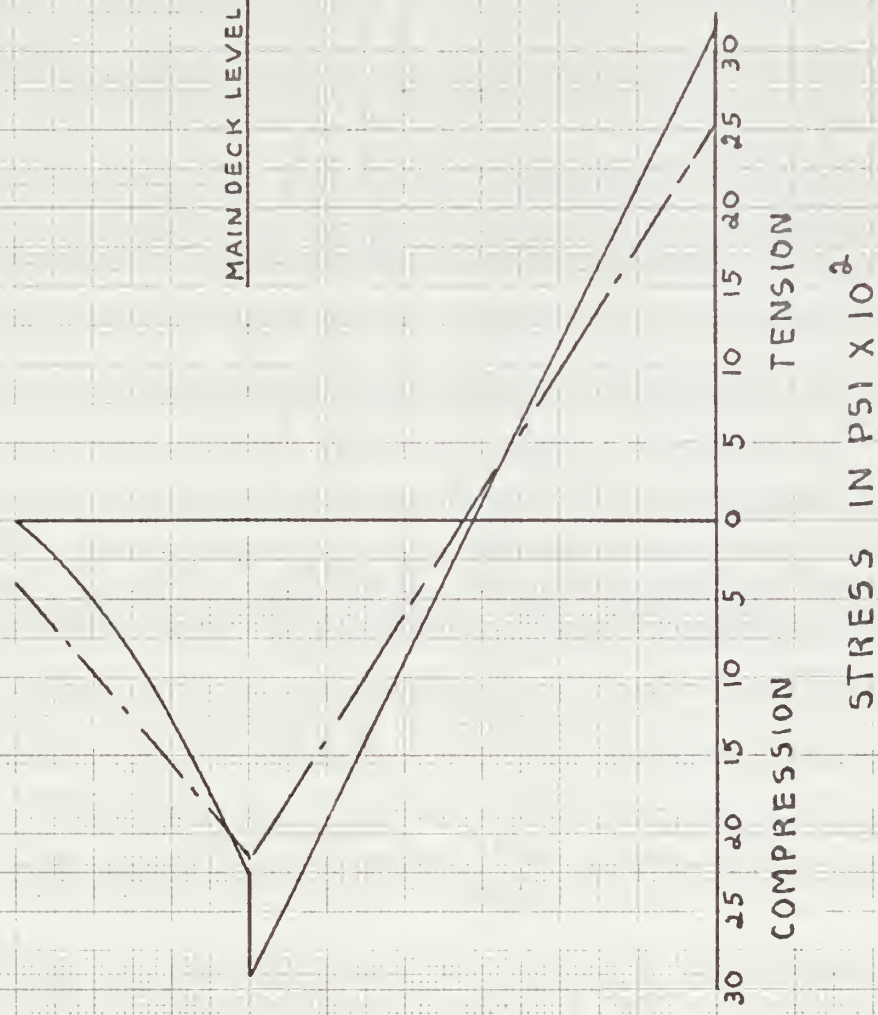
FIGURE 4.17



# COMPARISON OF METHODS

MODEL - WITH BULKHEADS  $l/B = 2.5$   
STRESS DIST. 35 FEET FROM AMIDSHIPS  
 $K = 1.53 \times 10^4$

— STRUDL  
--- BLEICH



MAIN DECK LEVEL

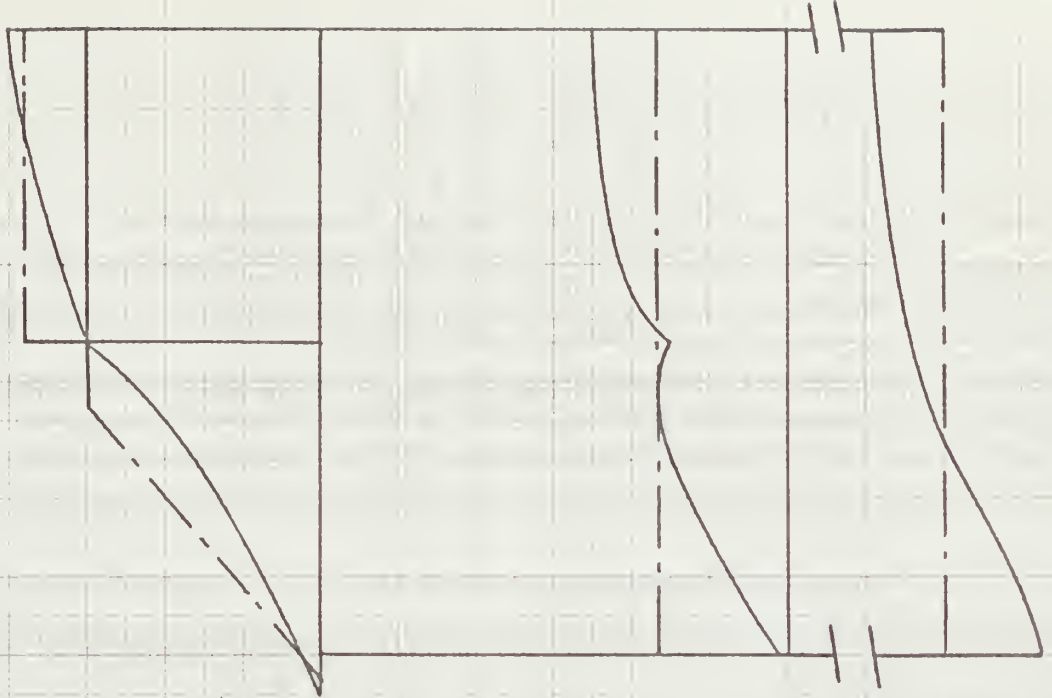


FIGURE 4.18



conditions; that is, the values of the moments obtained at each section corresponded to the respective moment on the bending moment diagram.

Because of the large shear-lag effects found in this particular model, it is difficult to make an accurate statement as to the applicability of linear superposition to Bleich's methods (which ignores shear-lag). It appears, however, that if these shear-lag effects were reduced, equilibrium would require that the STPUDL results approach Bleich's solution. This will be later checked through the use of a model whose increased  $l/B$  ratio significantly reduces the shear-lag.





## CHAPTER V - MODEL WITH INCREASED L/B

In this analysis the same model as presented in Chapter 4 was used except that all dimensions in the longitudinal direction were increased by a factor of two so as to increase the L/B ratio from 2.5 to 5.0. In order to maintain a bending-moment diagram similar to the one used in the preceding chapter, the distributive loading was reduced by a factor of four: i.e.,  $p_1 + p_2 = 1.4285$  tons/ft. From this loading  $S_C = S_D = 142.85$ , and  $M = M_C = M_D = 7,142.5$  ft. tons. Due to the increased distance between bulkheads, the value of the deck stiffness (K) was reduced to  $7.67 \times 10^3$  psi.

The same procedures as described in Chapter 4 were used in the determination of the STRUDL and Bleich solutions.

### 1. STRUDL Results

The longitudinal stress distributions at amidships and at a point 70 feet from amidships are shown in Figures 5.1 and 5.2, respectively. The most notable difference in these results from that of the preceding chapter is that the deckhouse has become more effective in the support of the stress distribution. This is due to the fact that the deckhouse length has been increased to 200 feet.

The discontinuity in the stress distribution across the main deck is still apparent but to a lesser degree. A look at the deflection curves of Figure 5.3 indicates that the differences in curvatures between the deckhouse connection and main deck at side are not as great as in the preceding model.





# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
ACROSS TRANSVERSE SECTION  
AT AMIDSHIPS  
MODEL - WITH BULKHEADS  
 $\lambda/B = 5.0$



FIGURE 5.1



# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
ACROSS TRANSVERSE SECTION  
70 FEET FROM AMIDSHIPS  
MODEL - WITH BULKHEADS  
 $\lambda/B = 5.0$

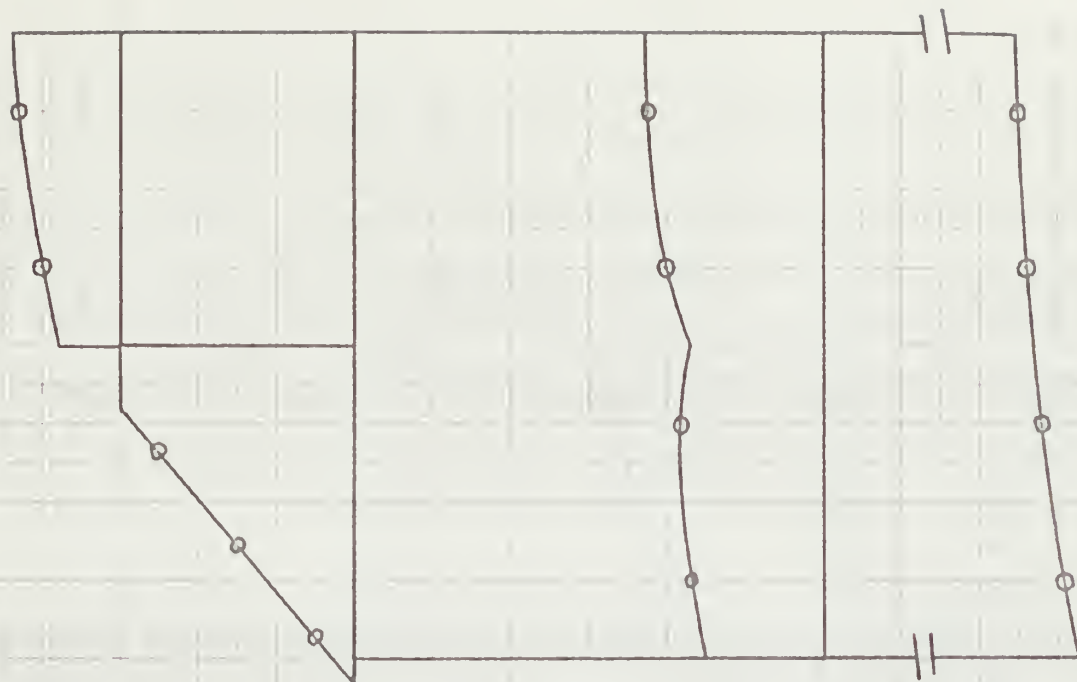
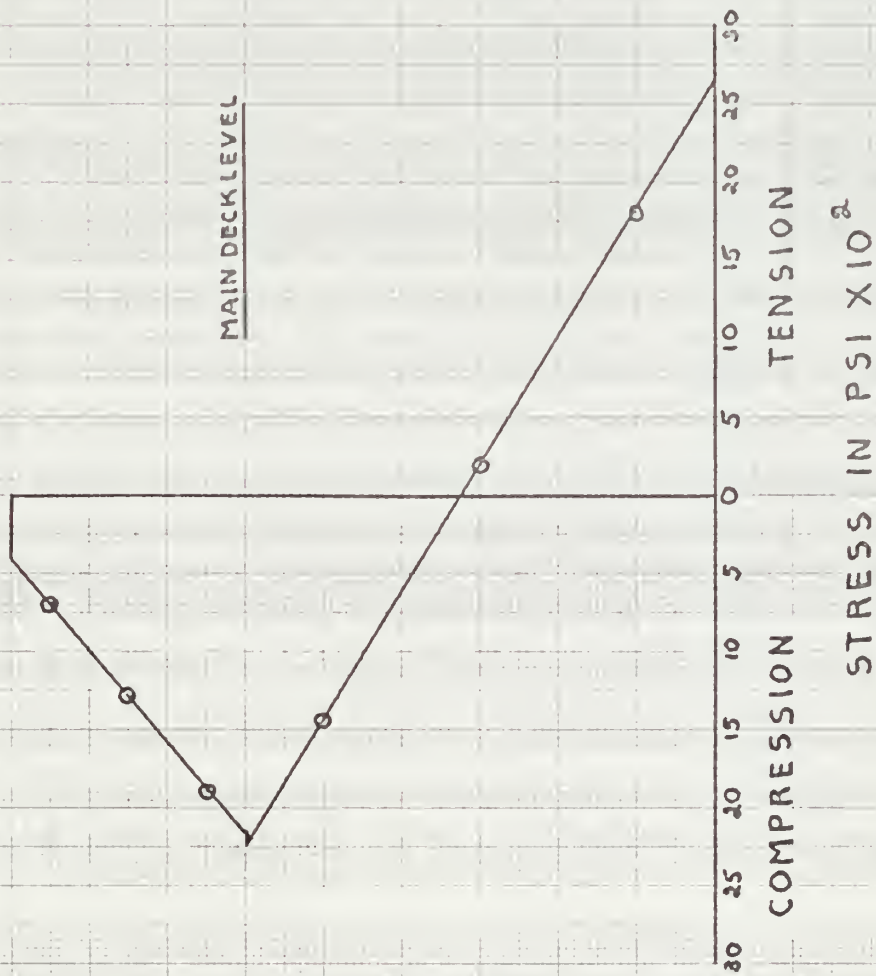


FIGURE 5.2





# STRUDL RESULTS

VERTICAL DEFLECTIONS OF  
MAIN DECK AT SIDE AND  
HULL-DECKHOUSE CONNECTION  
MODEL-WITH BHD'S  $R/B=5.0$

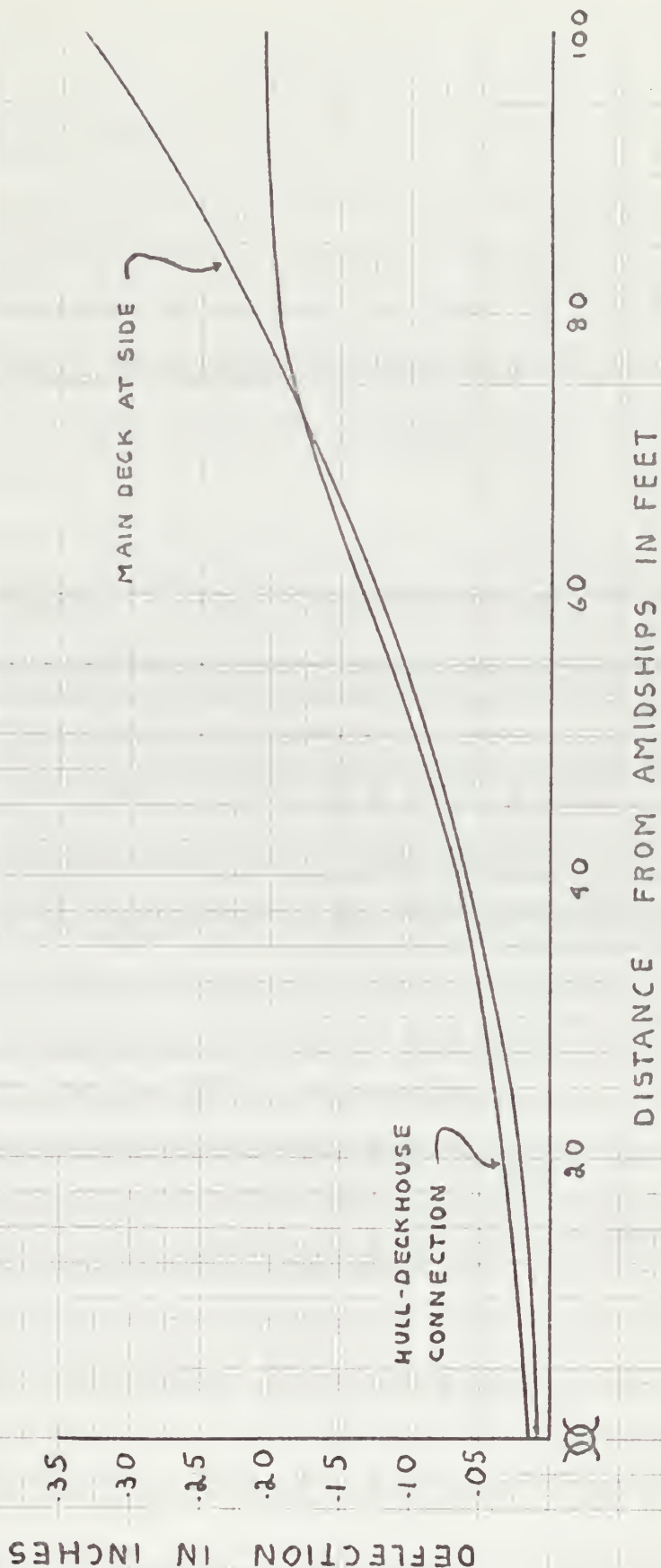


FIGURE 5.3



The most important result, however, is that the shear-lag effects have been reduced to a negligible amount.

## 2. Comparison of Methods

Comparison of STRUDL and Bleich results at amidships, 30 feet from amidships and 70 feet from amidships, are shown in Figure 5.4, 5.5, and 5.6, respectively. Again, the difference between solutions grows progressively worse as one travels away from amidships or the center of the deckhouse structure.

The most important result of this analysis is that at the amidships section (where Bleich's solution is most applicable), the two solutions are almost identical. The argument that linear superposition is applicable to Bleich's two general solutions is therefore confirmed by these results. It is readily evident that if the shear-lag effects were completely done away with, the two solutions would correlate exactly, except for the discontinuity at the hull-deckhouse connection. It must be kept in mind, however, that this says nothing of the applicability of Bleich's total solution for use as a design tool. In real life structures shear-lag effects are inevitably present to some degree as exemplified by all the STRUDL results thus far presented. Further discussions on this topic are presented in later sections after all model results have been presented.





# COMPARISON OF METHODS

MODEL-WITH BULKHEADS  $L/B = 5.0$

STRESS DIST. AT AMIDSHIPS

$K = 7.67 \times 10^3$

— STRUDL

--- BLEICH

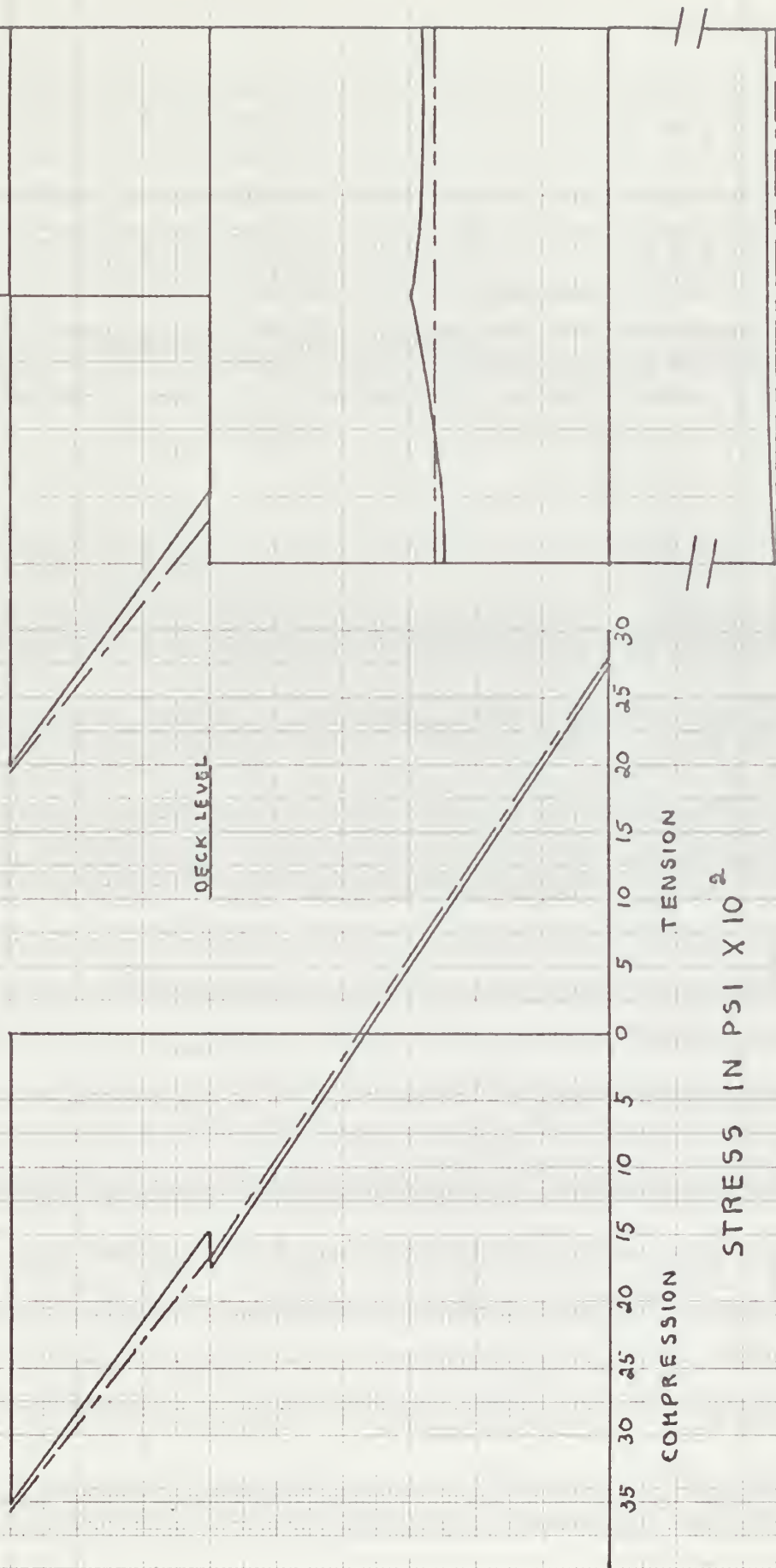


FIGURE 5.4



# COMPARISON OF METHODS

MODEL WITH BHD'S  $l/B = 5.0$   
 STRESS DIST. 30 FEET FROM AMIDSHIPS  
 $K = 7.67 \times 10^3$

— STRUDL  
 - - - BLEICH



FIGURE 5.5





# COMPARISON OF METHODS

MODEL - WITH BHD'S

$L/B = 5.0$

STRESS DIST. 70 FEET FROM AMIDSHIPS

$K = 7.67 \times 10^3$

— STRUDL

--- BLEICH

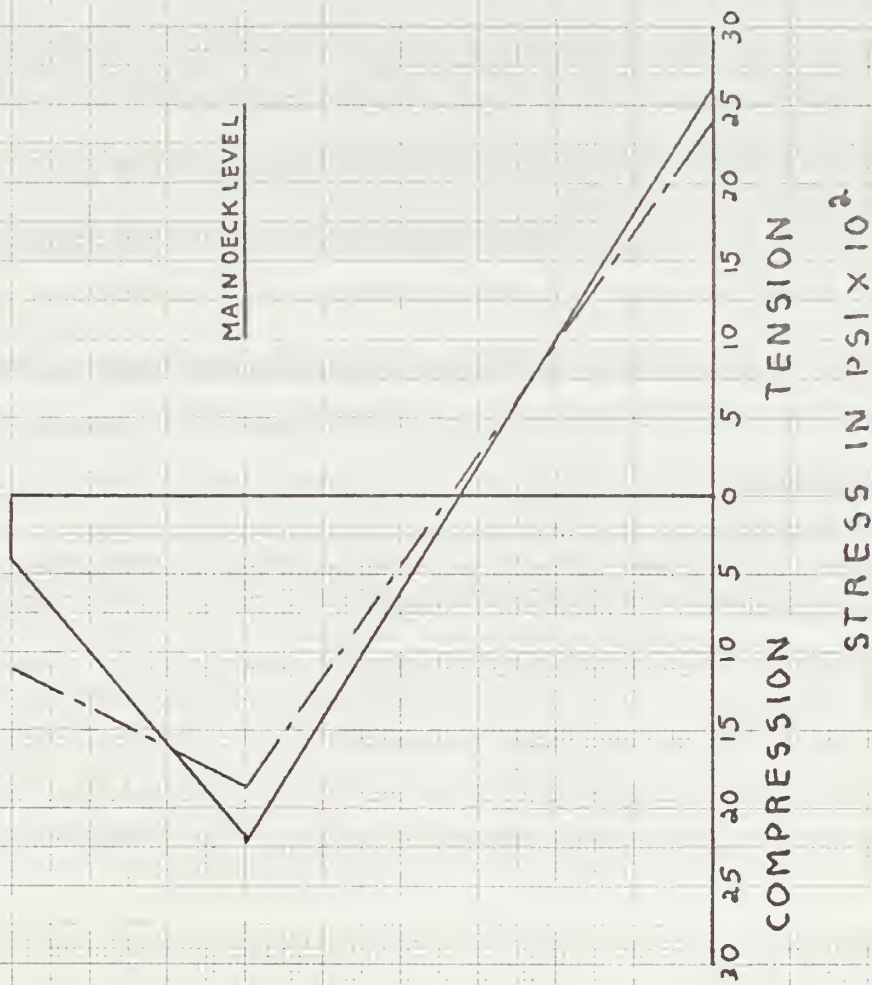


FIGURE 5.6



## CHAPTER VI - MODEL WITHOUT BULKHEADS

In order to investigate the effects on the stress distribution due to a more flexible deck, the bulkheads of the model presented in Chapter 4 were eliminated. All other parameters such as loading, lengths, boundary conditions, etc. remained unchanged.

For the STRUDL model, 'PSR' and 'BPP' elements were superimposed upon the whole structure so as to allow out-of-plane loads. Due to the nature of the imposed boundary conditions, which allows a considerable degree of flexibility, the program was not acceptable. It is also noted here that for models with bulkheads and using both 'PSR' and 'BPR' elements, the costs were prohibitive due to the increase in the number of elements. In order to obtain reliable results for the present analysis, only 'PSP' elements were used. The loads on the hull bottom were attached to transverse stiffeners in the same locations where the bulkheads had once been.

### 1. STRUDL Results

The longitudinal stress distributions at amidships and at a point 35 feet from amidships are shown in Figures 6.1 and 6.2, respectively. The most notable result is in the degree of stress reversal occurring along the hull and deckhouse sides. Because of the relatively flexible deck, the deckhouse has become less effective in the support of the stress distribution.

Shear-lag effects are as pronounced as in the original model; however, the discontinuities at the hull-deckhouse connections are not as great. Figure 6.3 supports the diminution of discontinuity





# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
ACROSS TRANSVERSE SECTION  
AT AMIDSHIPS  
MODEL - WITHOUT BULKHEADS  
 $L/B = 2.5$

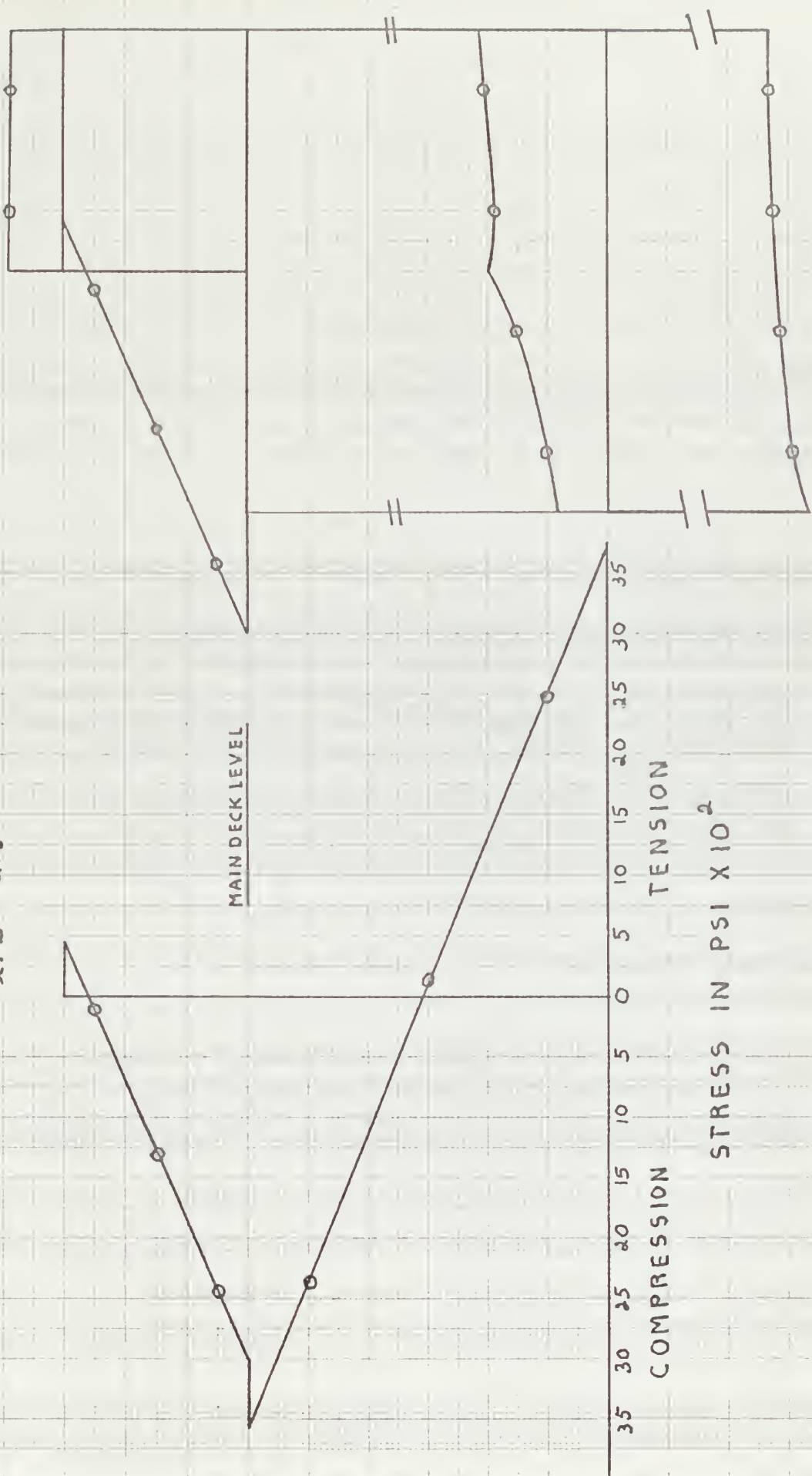


FIGURE 6.1



# STRUDL RESULTS

LONGITUDINAL STRESS DIST.  
35 FEET FROM AMIDSHIPS  
MODEL - WITHOUT BHD'S  
 $L/B = 2.5$

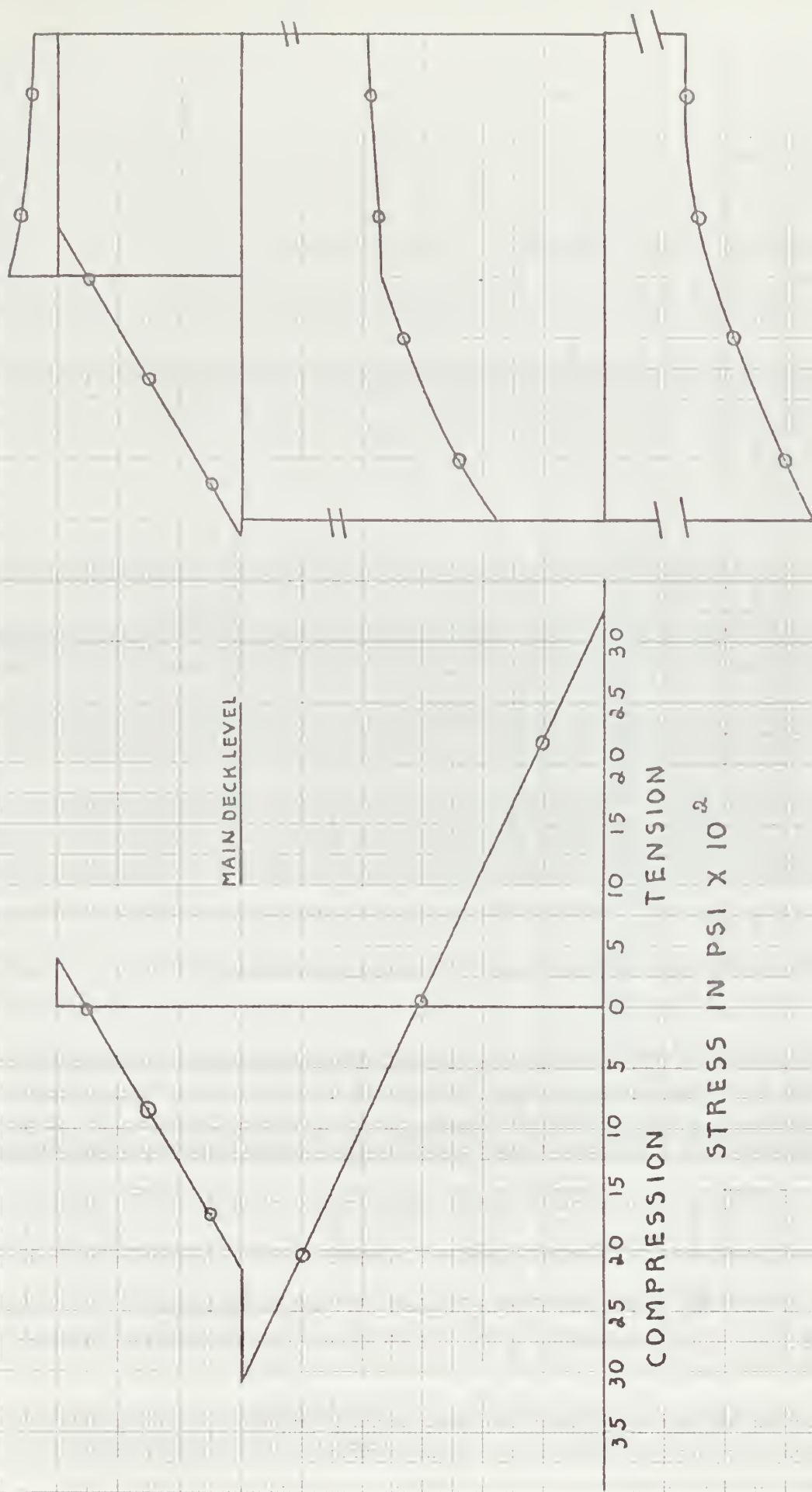


FIGURE 6.2





# STRUDL RESULTS

VERTICAL DEFLECTIONS OF  
MAIN DECK AT SIDE AND  
HULL-DECKHOUSE CONNECTION  
MODEL-WITHOUT BHD'S  $L/B=2.5$

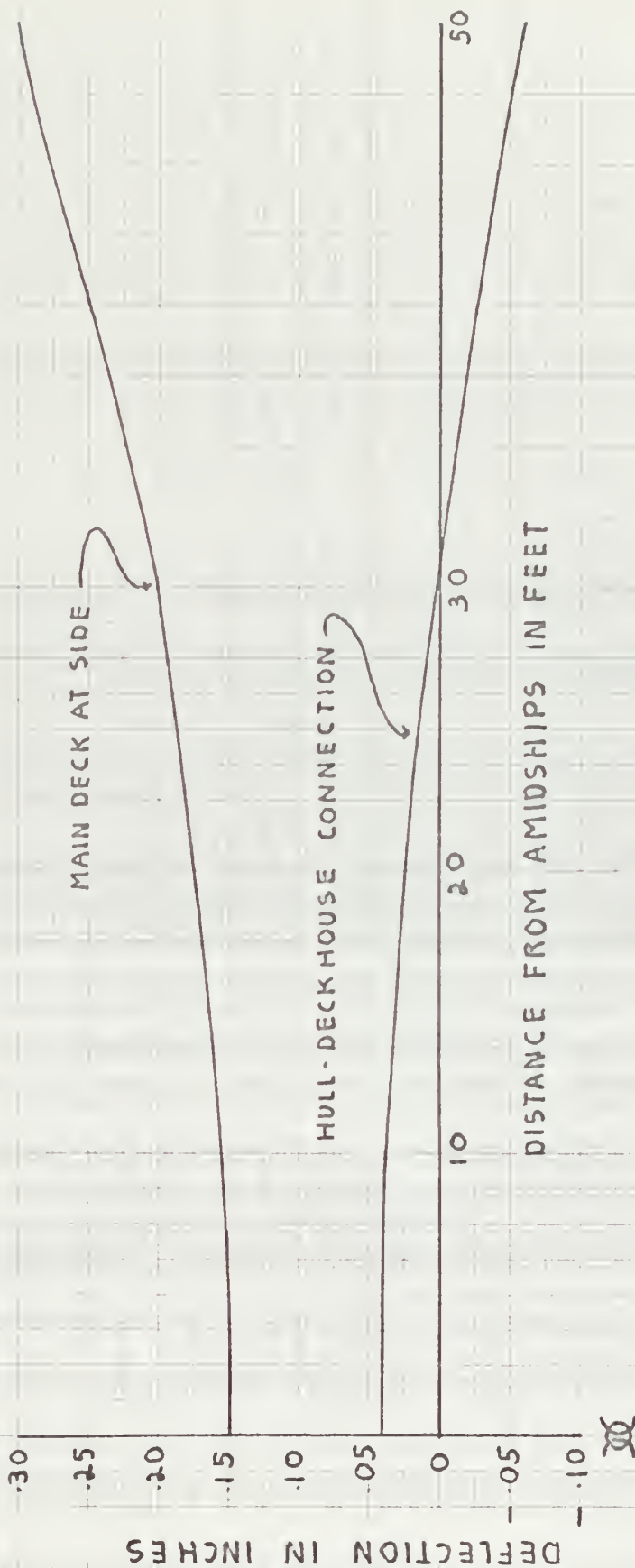


FIGURE 6.3



# STRUDL RESULTS

SHEAR STRESS DISTRIBUTION  
2.5 FEET ABOVE DECKHOUSE-  
HULL CONNECTION  
MODEL- WITHOUT BHD'S

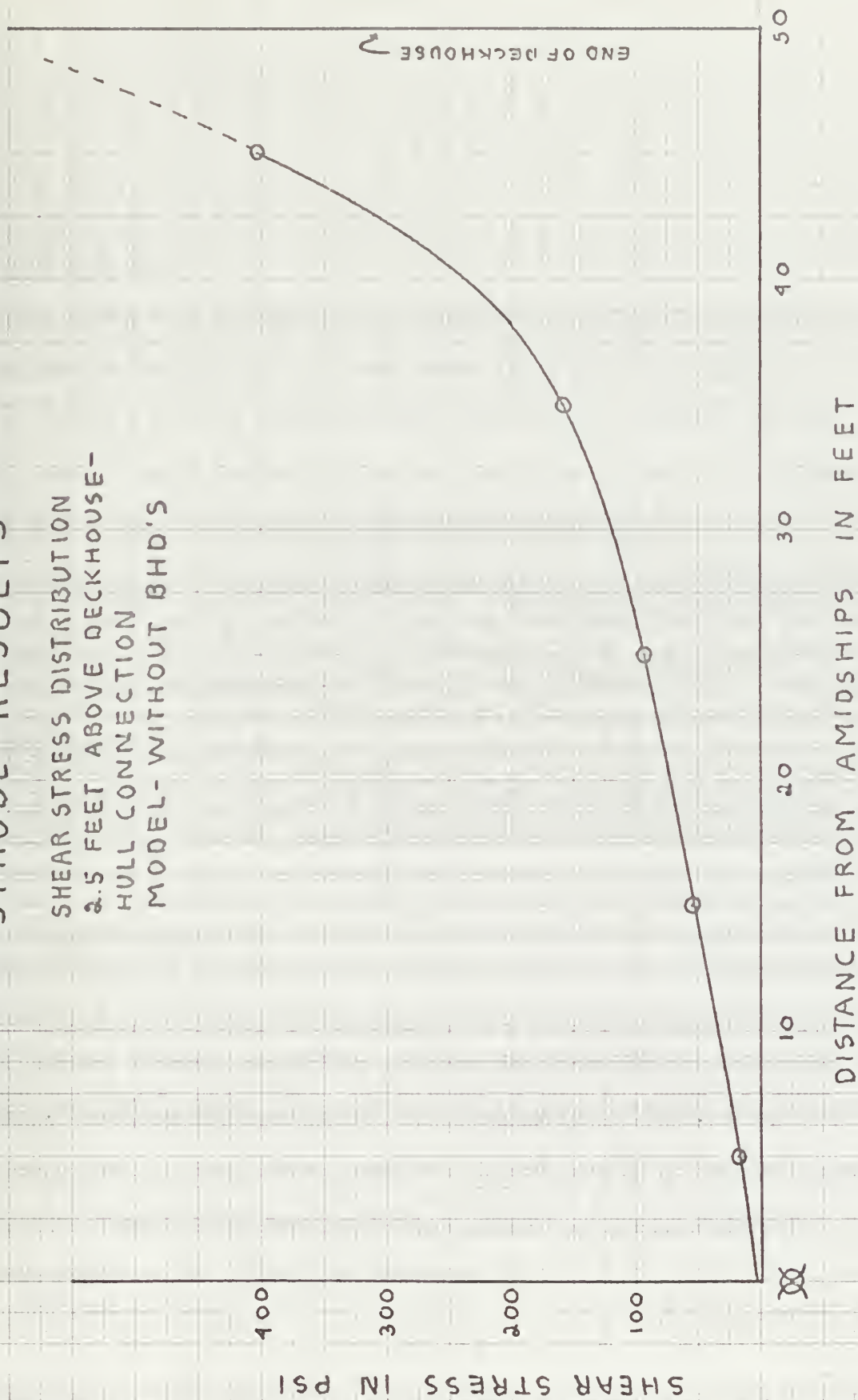


FIGURE 6.4





as well as highlighting the increase in relative vertical displacements between the hull-deckhouse connection and main deck at side.

The shear-stress distribution 2.5 feet above the hull-deckhouse connection is shown in Figure 6.4.

## 2. Bleich Model

The absence of bulkheads facilitates the analytical approach to the determination of  $K$ . A transverse slice of the main deck can be assumed to be a simple slender beam rigidly supported on its ends. This beam is subject to vertical deflections due to distributed loads ( $P$ ) so as to deflect the hull deckhouse connection 1 inch. Using simple-beam deflection theory, the value of  $K$  was found to be 1.736 psi. This value is very small when compared to the " for all other bulkhead models.

## 3. Comparison of Methods

Comparison of Bleich and STRUDL results at amidships, 15 feet from amidships and 35 feet from amidships, are shown in Figures 6.5, 6.6, and 6.7, respectively. Again, the difference between solutions grows progressively worse as one travels away from the amidships area. The Bleich distributions along the hull side produce lower peak values of stress as compared to STRUDL, whereas the distributions of stress along the deckhouse top and side are, in general, of greater magnitude than that of the STRUDL results. In each case the Bleich stress distribution across the main deck and hull bottom approximates an average value of the STRUDL distribution.



# COMPARISON OF METHODS

MODEL- WITHOUT BULKHEADS  $L/B = 2.5$

STRESS DIST. AT AMIDSHIPS

$K = 1.736$

— STRUOL  
- - - BLEICH

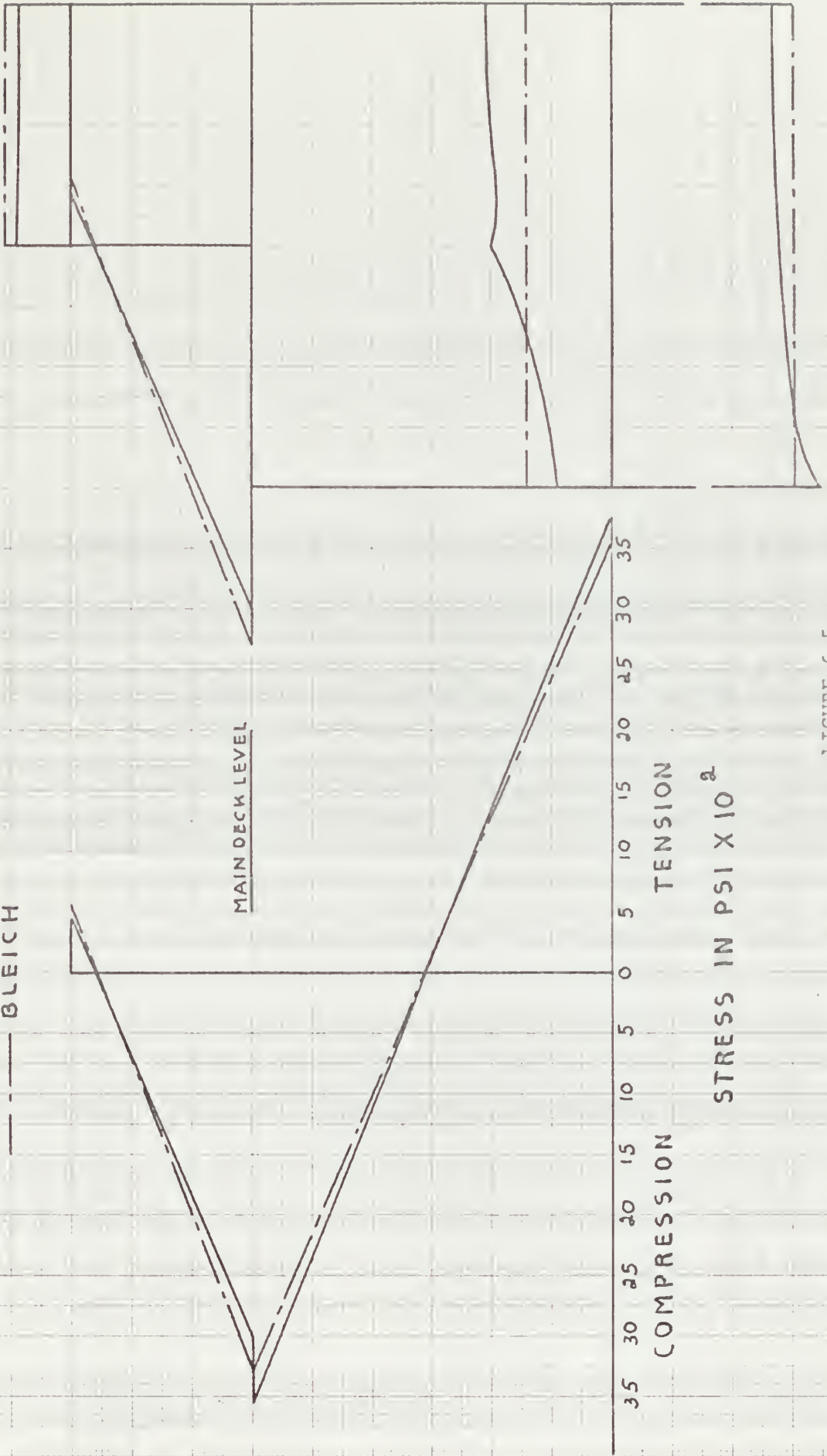


FIGURE 6.5



# COMPARISON OF METHODS

MODEL - WITHOUT BHD'S     $L/B = 2.5$   
STRESS DIST. 15 FEET FROM AMIDSHIPS  
 $K = 1.736$

— STRUDL  
- - - BLEICH

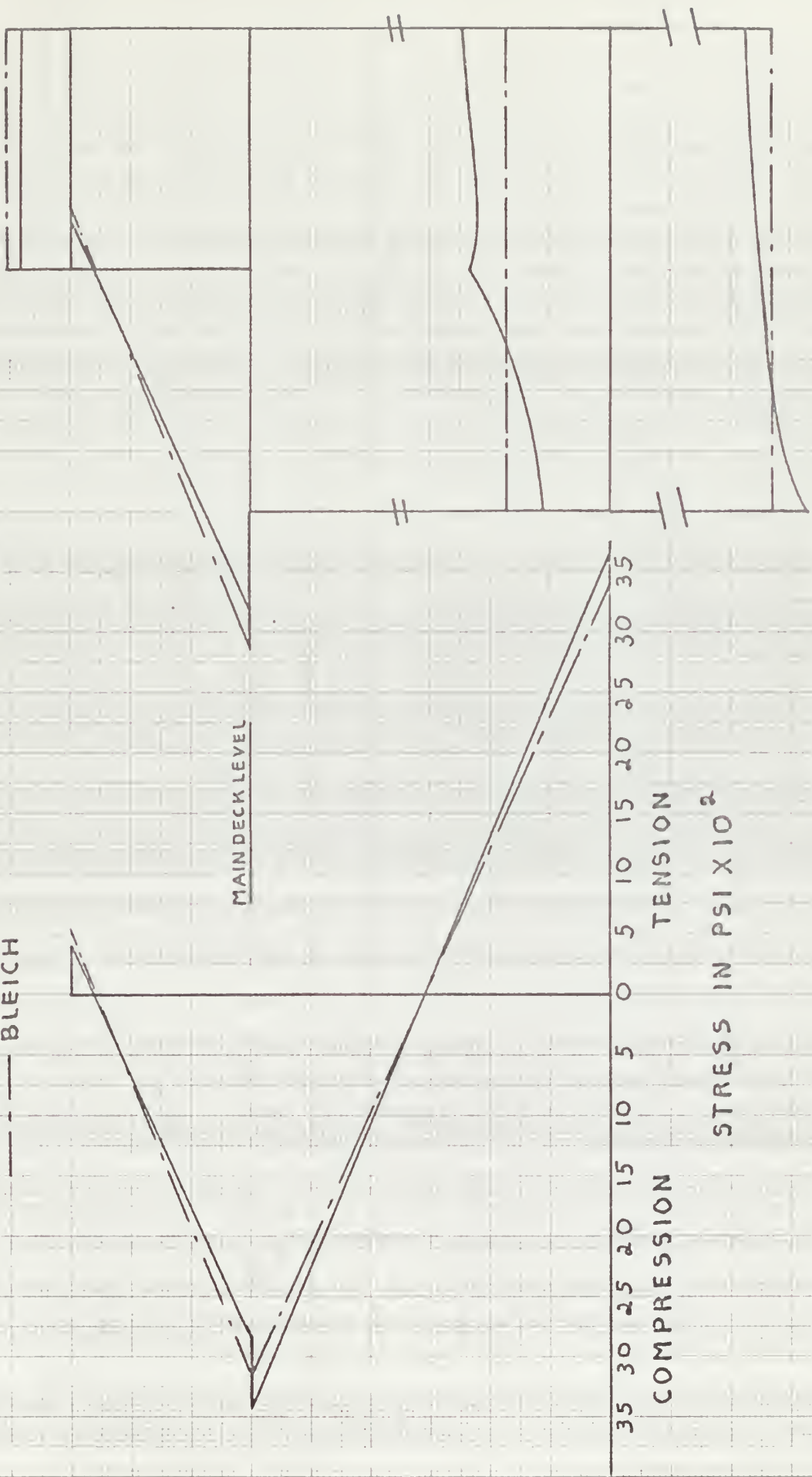


FIGURE 6.6





# COMPARISON OF METHODS

MODEL - WITHOUT BHD'S  $L/B = 2.5$   
 STRESS DIST. 35 FEET FROM AMIDSHIPS  
 $K = 1.736$

— STRUDL  
 - - - BLEICH

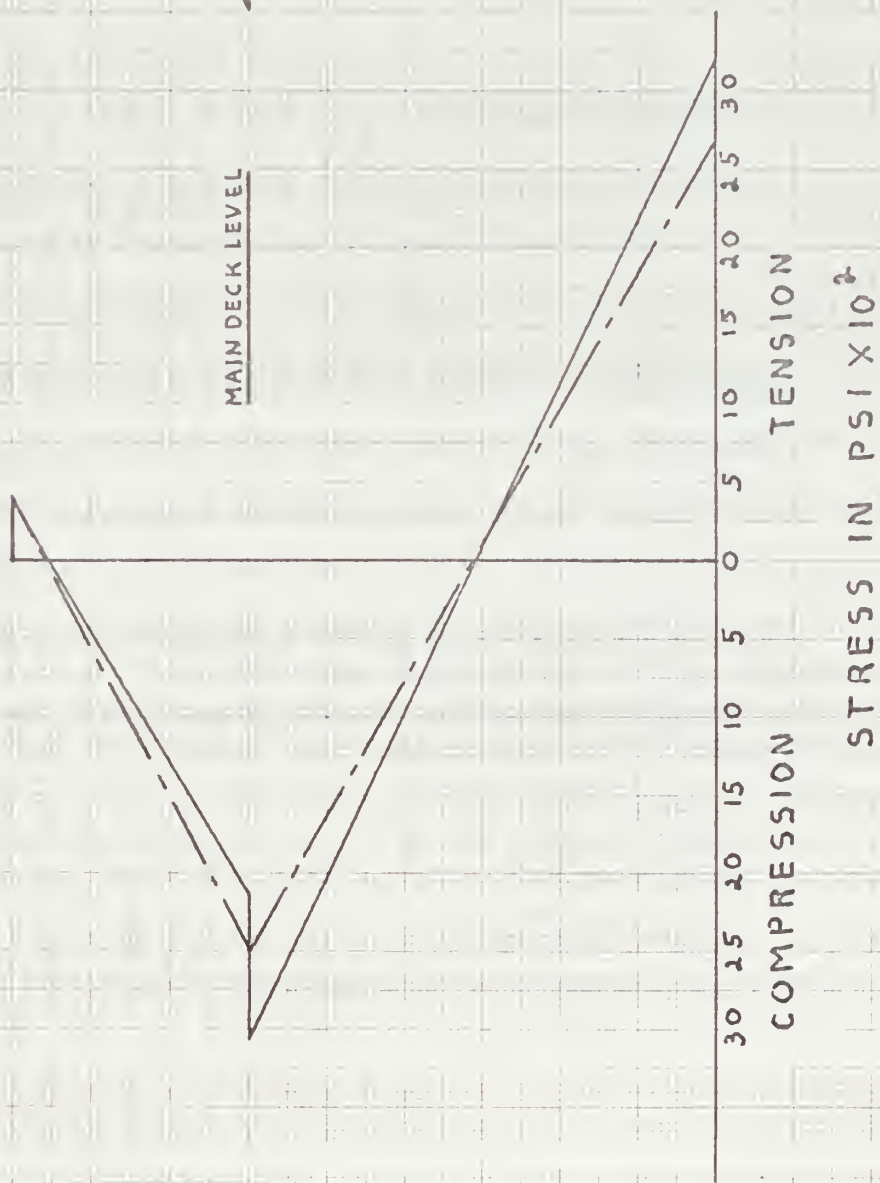


FIGURE 6.7





## CONCLUSIONS AND RECOMMENDATIONS

The principle of linear superposition can be applied to Bleich's two general solutions in order to obtain the total stress distribution solution. Because Bleich's method neglects shear-lag, the use of this method may be of limited significance when dealing with real-life structures. As the  $l/B$  ratio increases, however, Bleich's solution becomes more applicable for the center portions of the deckhouse-hull structure.

Because of the relatively favorable comparison of Bleich and STRUDL results in the center portions of the deckhouse-hull area for all models considered, there is some indication that Bleich methods may have some application for design purposes. The rather simple analytical methods which are employed for the determination of a stress solution is also a factor in consideration. All of the results of the present analysis indicate the Bleich method is conservative in reference to the stress distribution over the deckhouse proper and that it supplies an approximate average stress across the main deck. The only shortcoming is that the Bleich solution underestimates the value of the stress at the junctions of the hull side with the main deck and hull bottom. For example, in the model with bulkheads and  $l/B = 2.5$ , the value of the stress at the main deck at side is almost 20 per cent less than the STRUDL result. As the deckhouse length is increased so as to diminish the effects of the shear-lag, Bleich's solution converges towards the finite element solution, as evidenced in Figure 5.4. This supports the conclusion that Bleich's method is



more strongly applicable to structures having relatively long deck-houses.

In order to confirm the finite element results, especially in the area of the discontinuity occurring at the hull-deckhouse connection, it is recommended that a physical model be built similar to the mathematical models presented in this analysis. Strain and deflection analyses under similar loading conditions could then be used for verification, and augmentation of results obtained through STRUDL.



APPENDIX A - SAMPLE STRUDL PROGRAM

The following is a STRUDL program for finding element stresses and nodal displacements for a model with bulkheads spaced 20 feet apart.

```
STRUDL 'TEST 1' 'HULL-DECKHOUSE INTERACTION'
$ ONE QUARTER STRUCTURE
TYPE PLANE STRESS
DEBUG ALL
DUMP TIME
UNITS FEET
JOINT COORDINATES
1 0. 0. 0. S
2 5. 0. 0. S
3 10. 0. 0. S
4 15. 0. 0. S
5 20. 0. 0. S
6 0. 10. 0. S
7 5. 10. 0.
8 10. 10. 0.
9 15. 10. 0.
10 20. 10. 0.
11 0. 20. 0. S
12 5. 20. 0.
13 10. 20. 0.
14 15. 20. 0.
15 20. 20. 0.
16 0. 30. 0. S
17 5. 30. 0.
18 10. 30. 0.
19 15. 30. 0.
20 20. 30. 0.
21 0. 40. 0. S
22 5. 40. 0.
23 10. 40. 0.
24 15. 40. 0.
25 20. 40. 0.
26 0. 50. 0. S
27 5. 50. 0.
28 10. 50. 0.
29 15. 50. 0.
30 20. 50. 0.
31 20. 0. 10. S
32 20. 10. 10.
33 20. 20. 10.
```



34 20. 30. 10  
35 20. 40. 10  
36 20. 50. 10.  
37 20. 0. 20. S  
38 20. 10. 20.  
39 20. 20. 20.  
40 20. 30. 20.  
41 20. 40. 20.  
42 20. 50. 20.  
43 0. 0. 10. S  
44 5. 0. 10. S  
45 10. 0. 10. S  
46 15. 0. 10. S  
47 0. 20. 10. S  
48 5. 20. 10.  
49 10. 20. 10.  
50 15. 20. 10.  
51 0. 40. 10. S  
52 5. 40. 10.  
53 10. 40. 10.  
54 15. 40. 10.  
55 0. 0. 20. S  
56 5. 0. 20. S  
57 10. 0. 20. S  
58 15. 0. 20. S  
59 0. 20. 20. S  
60 5. 20. 20.  
61 10. 20. 20.  
62 15. 20. 20.  
63 0. 40. 20. S  
64 5. 40. 20.  
65 10. 40. 20.  
66 15. 40. 20.  
67 20. 0. 30. S  
68 20. 10. 30.  
69 20. 20. 30.  
70 20. 30. 30.  
71 20. 40. 30.  
72 20. 50. 30.  
73 15. 0. 30. S  
74 15. 10. 30.  
75 15. 20. 30.  
76 15. 30. 30.  
77 15. 40. 30.  
78 15. 50. 30.  
79 10. 0. 30. S  
80 10. 10. 30.  
81 10. 20. 30.  
82 10. 30. 30.  
83 10. 40. 30.  
84 10. 50. 30.  
85 5. 0. 30. S  
86 5. 10. 30.





87 5. 20. 30.  
88 5. 30. 30.  
89 5. 40. 30.  
90 5. 50. 30.  
91 0. 0. 30. S  
92 0. 10. 30. S  
93 0. 20. 30. S  
94 0. 30. 30. S  
95 0. 40. 30. S  
96 0. 50. 30. S  
97 0. 50. 35. S  
98 5. 50. 35.  
99 10. 50. 35.  
100 10. 40. 35.  
101 10. 30. 35.  
102 10. 20. 35.  
103 10. 10. 35.  
104 10. 0. 35. S  
105 0. 50. 40. S  
106 5. 50. 40.  
107 10. 50. 40.  
108 10. 40. 40.  
109 10. 30. 40.  
110 10. 20. 40.  
111 10. 10. 40.  
112 10. 0. 40. S  
113 10. 50. 45.  
114 10. 40. 45.  
115 10. 30. 45.  
116 10. 20. 45.  
117 10. 10. 45.  
118 10. 0. 45. S  
119 5. 50. 45.  
120 5. 40. 45.  
121 5. 30. 45.  
122 5. 20. 45.  
123 5. 10. 45.  
124 5. 0. 45. S  
125 0. 50. 45. S  
126 0. 40. 45. S  
127 0. 30. 45. S  
128 0. 20. 45. S  
129 0. 10. 45. S  
130 0. 0. 45. S

JOINT RELEASES FORCE Y Z MOMENT X

6 11 16 21 26 47 51 59 63 92 93 94 95 96 97 105 125 126 127 128 129

JOINT RELEASES FORCE X MOMENT Y

2 3 4 5 31 37 44 45 46 56 57 58 67 73 79 85 104 112 118 124

JOINT RELEASES FORCE Z

2 3 4 5 31 37 43 44 45 46 55 56 57 58 67 73 79 85 91 104 112 118 124 130

ELEMENT INCIDENCES

1 1 2 7 6



2 2 3 8 7  
3 3 4 9 8  
4 4 5 10 9  
5 6 7 12 11  
6 7 8 13 12  
7 8 9 14 13  
8 9 10 15 14  
9 11 12 17 16  
10 12 13 18 17  
11 13 14 19 18  
12 14 15 20 19  
13 16 17 22 21  
14 17 18 23 22  
15 18 19 24 23  
16 19 20 25 24  
17 21 22 27 26  
18 22 23 28 27  
19 23 24 29 28  
20 24 25 30 29  
21 46 31 5 4  
22 45 46 4 3  
23 44 45 3 2  
24 43 44 2 1  
25 58 37 31 46  
26 57 58 46 45  
27 56 57 45 44  
28 55 56 44 43  
29 73 67 37 58  
30 79 73 58 57  
31 85 79 57 56  
32 91 85 56 55  
33 50 33 15 14  
34 49 50 14 13  
35 48 49 13 12  
36 47 48 12 11  
37 62 39 33 50  
38 61 62 50 49  
39 60 61 49 48  
40 59 60 48 47  
41 75 69 39 62  
42 81 75 62 61  
43 87 81 61 60  
44 93 87 60 59  
45 54 35 25 24  
46 53 54 24 23  
47 52 53 23 22  
48 51 52 22 21  
49 66 41 35 54  
50 65 66 54 53  
51 64 65 53 52  
52 63 64 52 51  
53 77 71 41 66  
54 83 77 66 65  
55 89 83 65 64



56 95 89 64 63  
57 5 31 32 10  
58 10 32 33 15  
59 15 33 34 20  
60 20 34 35 25  
61 25 35 36 30  
62 31 37 38 32  
63 32 38 39 33  
64 33 39 40 34  
65 34 40 41 35  
66 35 41 42 36  
67 37 67 68 38  
68 38 68 69 39  
69 39 69 70 40  
70 40 70 71 41  
71 41 71 72 42  
72 73 67 68 74  
73 74 68 69 75  
74 75 69 70 76  
75 76 70 71 77  
76 77 71 72 78  
77 79 73 74 80  
78 80 74 75 81  
79 81 75 76 82  
80 82 76 77 83  
81 83 77 78 84  
82 85 79 70 86  
83 86 80 81 87  
84 87 81 82 88  
85 88 82 83 89  
86 89 83 84 90  
87 91 85 86 92  
88 92 86 87 93  
89 93 87 88 94  
90 94 88 89 95  
91 95 89 90 96  
92 72 104 103 80  
93 80 103 102 81  
94 81 102 101 82  
95 82 101 100 83  
96 83 100 99 84  
97 99 90 94 80  
98 97 98 90 96  
99 104 112 111 103  
100 103 111 110 102  
101 102 110 109 101  
102 101 109 108 100  
103 100 108 107 99  
104 106 107 99 98  
105 105 106 98 97  
106 112 118 117 111  
107 111 117 116 110  
108 110 116 115 114



109 109 115 114 108  
110 108 114 113 107  
111 119 113 107 106  
112 125 119 106 105  
113 124 118 117 123  
114 123 117 116 122  
115 122 116 115 121  
116 121 115 114 120  
117 120 114 113 119  
118 130 124 123 129  
119 129 123 122 128  
120 128 122 121 127  
121 127 121 120 126  
122 126 120 119 125

UNITS INCHES POUNDS

MOMENT PROPERTY C

1 TO 29 TYPE 'PSR', THICKNESS 0.50  
21 TO 32 TYPE 'PSR', THICKNESS 0.125  
33 TO 44 TYPE 'PSR', THICKNESS 0.25  
45 TO 56 TYPE 'PSR', THICKNESS 0.25  
57 TO 71 TYPE 'PSR', THICKNESS 0.50  
72 TO 91 TYPE 'PSR', THICKNESS 0.50  
92 TO 122 TYPE 'PSR', THICKNESS 0.25

CONSTANTS

E 30000000. ALL

POISSON 0.3 ALL

G 11538462. ALL

UNITS KIPS

LOADING 'TOTAL'

\$ TOTAL LOAD INCLUDES SHEAR, MOMENT AND PRESSURE

JOINT LOADS

\$ PRESSURE LOADS

5 FORCE Z -8.0

2 3 4 11 15 21 25 FORCE Z -16.0

12 13 14 22 23 24 FORCE Z -32.0

\$ SHEAR AND MOMENT LOADS

26 FORCE Y 25.6672

27 28 29 FORCE Y 53.3344

30 FORCE Y 68.1072 Z 49.28

36 FORCE Y 35.5936 Z 110.712

42 FORCE Y -35.5936 Z 110.712

72 FORCE Y -68.1072 Z 49.28

78 84 90 FORCE Y -53.3344

96 FORCE Y -26.6672

UNITS FEET KIPS

PRINT DATA

STIFFNESS ANALYSIS

UNITS INCHES POUNDS

LIST DISPLACEMENTS STRESSES ALL

FINISH





## APPENDIX B - CALCULATIONS FOR DETERMINING K

The following is an analytical approach to the determination of the spring constant (K) for a model with bulkheads spaced 20 feet apart..

### Basic Nomenclature

- A - area of beam cross section
- K' - factor depending on shape of beam cross section
- P - distributed load at hull-deckhouse connection
- V - vertical shear due to actual forces
- v - vertical shear due to load of one pound acting at the section where the deflection is to be determined
- $Y_M$  - deflection due to internal moments
- $Y_T$  - deflection due to shear
- $Y_T$  - total deflection
- x - distance along length of beam
- < > - indicates singularity functions

### Deflection Due to Moment Only

$$EI \frac{d^2 Y_M}{dx^2} = 240[P_x - P \langle x - 120 \rangle^1 - P \langle x - 360 \rangle^1]$$

$$EI Y_M = 240P \left[ \frac{x^3}{6} - \frac{\langle x - 120 \rangle^3}{6} - \frac{\langle x - 360 \rangle^3}{6} + C_1 x + C_2 \right]$$

$$B. C. \quad Y_M = 0 \quad \text{when} \quad x = 0, \quad 480''$$

$$E = 30 \times 10^6 \text{ psi}$$

$$I = 8,748,005 \text{ in}^4$$



$$Y_M = - \frac{(2.105 \times 10^{-5})P}{1} \text{ at } x = 120''$$

### Deflection Due to Shear

$$Y_T = - \frac{1}{K'} \int \frac{V_v}{AG} dx \text{ (method of unit loads)}$$

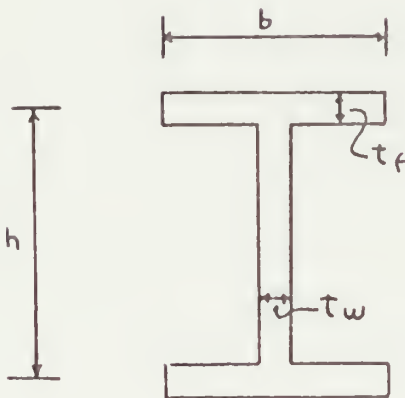
$$Y_T = - \frac{1}{K'} \frac{P(240)x}{AG}$$

$$K' = \frac{10(1 + \gamma)(1 + 3m)^2}{6(2 + 12m + 25m^2 + 15m^3) + \gamma(11 + 66m + 135m^2 + 90m^3) + 30mn^2(1 + m) + 5\gamma mn^2(8 + 9m)}$$

where

$$m = \frac{2b t_f}{h t_w} \quad n = \frac{b}{h}$$

I BEAM CROSS SECTION



$$t_f = 0.5'' ; t_w = .25'' ; h = 360'' , b = 240''$$

$$m = 2.66 ; n = .666$$

$$K' = \underline{.262}$$

It is customary to assume that only webs of structural shapes such as channels and I beams resist shearing stresses because shear stresses are small in flanges.

$$Y_T = - \frac{(10.55 \times 10^{-5})P}{1}$$

$$Y_T = Y_T + Y_M = 1''$$



$$l'' = - p(10.76 \times 10^{-5})$$

$$p = .93 \times 10^4$$

$$K = 2p$$

$$\underline{K = 1.86 \times 10^4 \text{ psi}}$$



## APPENDIX C - SAMPLE BLEICH CALCULATIONS

The following is a solution for the stress distribution at amidships for a model with bulkheads spaced 20 feet apart.

### Basic Nomenclature

- a - distance between centers of gravity of hull and deckhouse sections
- $A_1$  - cross sectional area of deckhouse
- $A_2$  - cross sectional area of hull
- I - total moment of inertia of structure cross section
- $I_A$  - factor for determining I
- $I_1$  - moment of inertia of deckhouse cross section
- $I_2$  - moment of inertia of hull cross section
- K - spring constant
- l - length of deckhouse
- M - constant moment part of loading
- $M_p$  - moment in midship section due to the loads  $p_1 + p_2$
- $\Delta M_1$  - correction of direct force acting on deckhouse
- $\Delta M_2$  - correction of direct force acting on hull
- $\Delta M_1$  - correction of moment acting on deckhouse
- $\Delta M_2$  - correction of moment acting on hull
- $(p_1 + p_2)$  - equally distributed loads
- u - parameter described by dimensions of hull and deckhouse and of stiffness factor K
- $x_1$  - vertical distance from center of gravity of deckhouse cross section





- $x_2$  - vertical distance from center of gravity of hull cross section
- $x$  - vertical distance from center of gravity of entire cross section
- $\alpha_1$  - ratio of the distance of center of gravity of deckhouse from main deck
- $\alpha_2$  - ratio of the distance of center of gravity of hull from main deck
- $\gamma$  - parameter described by dimensions of hull and deckhouse and of stiffness factor  $K$
- $\mu$  - size factor; measure of the size of the deckhouse in relation to hull
- $\sigma_M$  - stress solution for constant moment
- $\sigma_N$  - Navier's stress
- $\sigma_P$  - stress solution for distributed load
- $\sigma_T$  - total stress solution
- $\Delta\sigma_1$  - corrective stress for deckhouse
- $\Delta\sigma_2$  - corrective stress for hull
- $\phi_1$  - deviation factor used in constant moment solution
- $\phi_2$  - deviation factor used in distributed loading solution

Significant Values of Mathematical Model

$$\begin{aligned}a &= 306'' \quad A_1 = 150 \text{ in}^2 \quad A_2 = 840 \text{ in}^2 \\I_1 &= 534,600 \text{ in}^4 \quad I_2 = 19,440,000 \text{ in}^4 \\M &= M_P = 192 \times 10^6 \text{ in lbs} \\\alpha_1 &= .4118 \quad \alpha_2 = .5882 \\K &= 1.53 \times 10^4 \text{ psi}\end{aligned}$$



Bloch Calculations

$$I_A = a^2 \frac{A_1 A_2}{A_1 + A_2} = 11,917,309 \text{ in}^4$$

$$I = I_1 + I_2 + I_A = 31,801,909 \text{ in}^4$$

$$e = \frac{I_1 + \alpha_1 I_A}{I_2 + \alpha_2 I_A} = .206$$

Because in this model  $M = M_p$ ,  $(\Delta S)_M = (\Delta S)_p$

$$\Delta M_1 = \frac{\mu I_A}{a} \frac{\mu(1 - \mu)}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} = - \Delta M_2$$

$$\Delta M_1 = 18.9 \times 10^4 \text{ lbs.}$$

$$\Delta M_1 = - M I_1 \frac{\mu}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} = - 8.93 \times 10^6 \text{ in lbs.}$$

$$\Delta M_2 = M I_2 \frac{\mu^2}{(1 + \mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} = 6.59 \times 10^7 \text{ in lbs.}$$

$$\Delta \sigma_1 = \frac{\Delta M_1}{A_1} - \frac{\Delta M_1}{I_1} x_1$$

$$\Delta \sigma_1 = 1200 + 16.7 x_1$$

$$\Delta \sigma_1 \text{ (deckhouse top)} = 2167 \text{ psi ; } x_1 = 54''$$

$$\Delta \sigma_1 \text{ (main deck)} = - 844 \text{ psi ; } x_1 = - 126''$$

$$\Delta \sigma_2 = \frac{\Delta M_2}{A_2} - \frac{\Delta M_2}{I_2} x_2$$

$$\Delta \sigma_2 = - 225 - 3.44 x_2$$

$$\Delta \sigma_2 \text{ (main deck)} = - 844 \text{ psi ; } x_2 = 180''$$

$$\Delta \sigma_2 \text{ (hull bottom)} = 394 \text{ psi ; } x_2 = - 180''$$



$$u = \frac{\gamma \ell}{2} = \frac{\gamma \ell}{2} = \frac{\ell}{2} \sqrt{\frac{1 + \mu}{a_2 I_1 + \mu a_1 I_2}}$$

$$u = 1.735$$

$$\text{from graphs } \phi_1 = .274, \quad \phi_2 = .355$$

$$\sigma_N = - \frac{(W + M)}{I} x$$

$$\sigma_N \text{ (deckhouse top)} = - 3770 \text{ psi} : x = 313.64''$$

$$\sigma_N \text{ (main deck)} = - 1600 \text{ psi} : x = 133.64''$$

$$\sigma_N \text{ (hull bottom)} = 2720 \text{ psi} : x = -226.36''$$

$$\sigma_T = \sigma_N + \sigma_p$$

$$\sigma_T = \sigma_N + \phi_1(z) (\Delta\sigma)_N + \phi_2(z) (\Delta\sigma)_p$$

$$\text{at } z = 0 \quad \phi_1(z) = \phi_1 : \quad \phi_2(z) = \phi_2$$

#### Final Solution

$$\sigma_T \text{ (deckhouse top)} = - 2390 \text{ psi}$$

$$\sigma_T \text{ (main deck)} = - 2167 \text{ psi}$$

$$\sigma_T \text{ (hull bottom)} = 2000 \text{ psi}$$



BIBLIOGRAPHY

1. Bleich, H. H., "Non-Linear Distribution of Bending Stresses Due to Distortion of the Cross Section," J. of Appl. Mech., vol. 20, 1953, p. 95.
2. Buhler, J. and Levy, L. J., "Experimental and Theoretical Studies of Hull-Deckhouse Interaction- 2 - Level Deckhouse," Office of Naval Ship Research, U. S. Navy, August, 1971.
3. Caldwell, J. P., "The Effect of Superstructures on the Longitudinal Strength of Ship," Transactions, I.N.A., vol. 90, 1957, p. 664.
4. Chapman, J. C., "The Interaction Between a Ship's Hull and a Long Structure," Transactions, I.N.A., vol. 90, 1957, p. 618.
5. Comstock, J. P., Principles of Naval Architecture, S.N.A.M.E., New York, N. Y., 1967.
6. D'Arcangelo, A. M., Ship Design and Construction, S.N.A.M.E., New York, N. Y., 1969.
7. Hopkins, B. R., Design Analysis of Shafts and Teams, McGraw-Hill, New York, N. Y., 1970.
8. Johnson, A. J., "Stresses in Deckhouses and Superstructures," Transactions, I.N.A., vol. 90, 1957, p. 634.
9. Logcher, R. D., et al., ICES STRUDL II, Structural Design Language Engineering User's Manual, vol. 1 and 2, Department of Civil Engineering, M.I.T., Cambridge, Mass., 1971.
10. Shade, H. A., "Two-Beam Deckhouse Theory with Shear Effects," Dept. of Navy, BU Ships Report No. NA-65-3, October, 1965.









Thesis  
R6716

**134739**  
Rodrigues  
The application of  
linear superposition  
to Bleich's two-beam  
theory.  
5 SEP 72  
DISPLAY

Thesis  
R6716

Rodrigues

The application of  
linear superposition  
to Bleich's two-beam  
theory.

**134739**

thesR6716

The application of linear superposition



3 2768 001 94940 7

DUDLEY KNOX LIBRARY